Extended Abstract 049

Title:
Computing Mitered Offset Curves Based on Straight Skeletons

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Introduction:

The straight skeleton of a polygon in 2D was first defined by Aichholzer et al. [2]. It is the geometric graph whose edges are the traces of vertices of shrinking mitered offset curves of the polygon, see Figure 1, left. Straight skeletons are a versatile tool in computational geometry and have found applications in diverse fields of industry and science. E.g., Tomoeda et al. use straight skeletons to create signs with an illusion of depth [8], while Sugihara uses (weighted) straight skeletons in the design of pop-up cards [6]. Aichholzer et al. [2] apply straight skeletons for roof design and terrain generation.

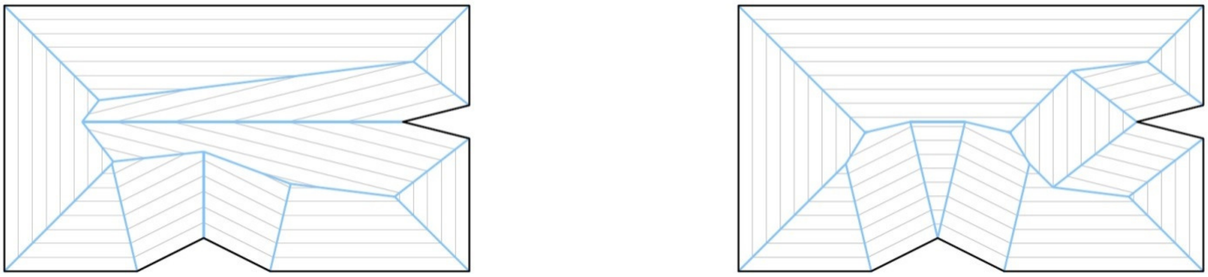

Fig. 1: (Left) The straight skeleton *S*(*P*) (blue) of an input polygon *P* (bold) is the union of the traces of offset-curve vertices. Several mitered offset curves are shown in gray. (Right) Beveled offset curves based on the linear axis, a variant of the straight skeleton.

Prior Work:

Almost 20 years ago, Aichholzer and Aurenhammer presented an algorithm to compute straight skeletons [1]. Their algorithm simulates the so-called wavefront propagation, where an initial mitered offset curve with offset zero is created. This initial wavefront is identical to the input polygon. Its edges then propagate towards the polygon's interior (or exterior) at unit speed in a self-parallel manner. During this propagation process, topological changes occur in the wavefront: (i) edges vanish when they shrink to zero length, and (ii) edges may be split into two parts when another portion of the wavefront crashes into them.

Aichholzer and Aurenhammer [1] maintain a kinetic triangulation of the interior of the offset polygon(s) during the wavefront propagation process. All topological changes are witnessed by the collapse of a triangle of the kinetic triangulation. (That is, its area becomes zero.) The basic idea is to keep track of the collapse times of all kinetic triangles in order to know the time of the next topological change of the wavefront. Their algorithm is easily extended to arbitrary collections of line segments that do not intersect except at common end points, so-called planar straight-line graphs (PSLGs).

Our Contribution:

In order to provide a practical tool for mitered offsetting, we converted Aichholzer and Aurenhammer's theoretical description into an implementation that can cope with real-world data. In Palfrader et al. [5], we sketched the theoretical basis of an extension and modification of their algorithm necessary for computing the straight skeleton of a general PSLG within the entire plane, without relying on an implicit assumption of general position of the input.

More recently, while implementing our straight-skeleton code Surfer based on the theoretical basis laid out in [5], we investigated the peculiarities of a realization of that algorithm on a standard floating-point arithmetic. For instance, we refined the naive (determinant-based) computation of the collapse times in order to make it numerically more reliable.

In addition to constructing the straight skeleton, Surfer is able to compute mitered offset curves: Surfer maintains a data structure while computing the straight skeleton which then can be used to quickly compute offset curves for any desired offset, by iterating over all faces of the straight skeleton. Note that straight-skeleton based offsetting does not require time-consuming operations like computing pairwise intersections or removing excess loops of the offsets. And, of course, the offset need not be known prior to computing the straight skeleton itself. Given the straight skeleton, one mitered offset curve of a polygon with 100000 vertices can be computed in about 10 ms using a 2010 Intel Core i7-980X CPU clocked at 3.33 GHz.

It is well known that mitered offset intersections (for acute angles at reflex vertices) can be far away from their defining input. Hence, we extended our straight-skeleton based approach to support linear-axis [7] based offsetting, resulting in beveled offsets. Thus, the distance between any offset curve point and its input is bounded by $\sqrt{2}$ times the parallel offsetting distance. See Figure 1, right. Our algorithm can also compute multi-segment bevels.

We tested Surfer on about twenty thousand polygons and PSLGs, with up to 2.5 million vertices per input. Both real-world and contrived data of different characteristics was tested. Some datasets contain also circular arcs, which we approximated by polygonal chains in a preprocessing step. Our tests show that Surfer runs in $O(n \log n)$ time and linear space for all practical input. From a theoretical point of view, one can design contrived inputs that cause the algorithm to consume more time, but we have not encountered any such input in our tests. Surfer can work with both standard IEEE 754 double precision arithmetic as well as arbitrary-precision arithmetic using the MPFR library [4]. Runtime and memory usage of Surfer in its IEEE 754 double-precision mode is plotted in Figure 2. Measurements were conducted on a 2010 Intel Core i7-980X CPU clocked at 3.33 GHz.

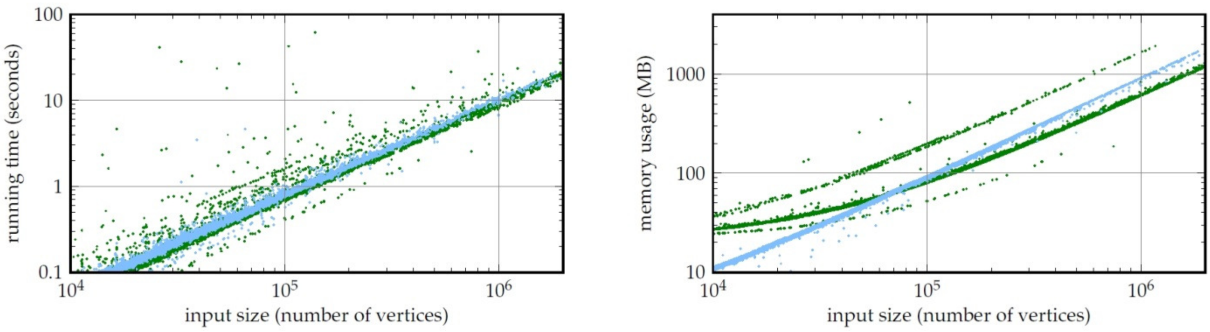

Fig. 2: Runtime and memory usage behavior of Surfer for inputs of different sizes (*x*-axis) in blue. Its performance characteristics are similar to Held's code VRONI [3], plotted in green, which generates conventional (rounded) offsets based on Voronoi diagrams of the input.

References:

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