

Straight Skeleton Implementations based on Exact Arithmetic

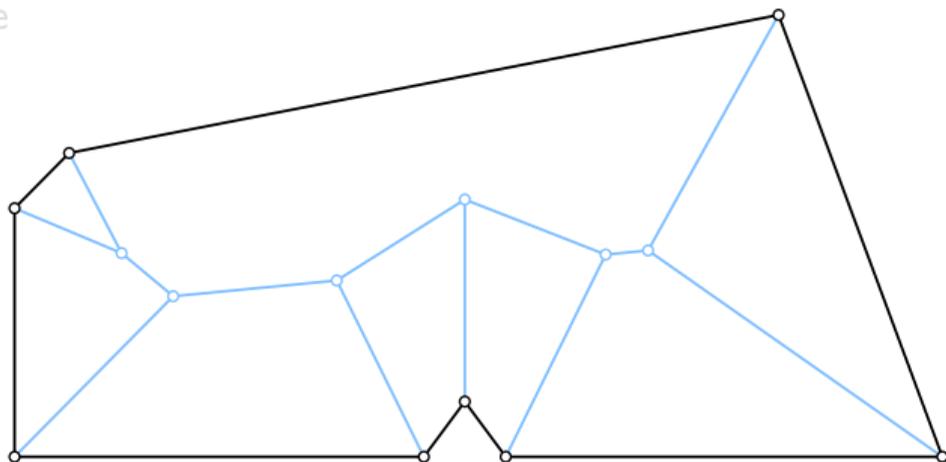
Günther Eder, Martin Held, and Peter Palfrader



Online Conference, March 2020

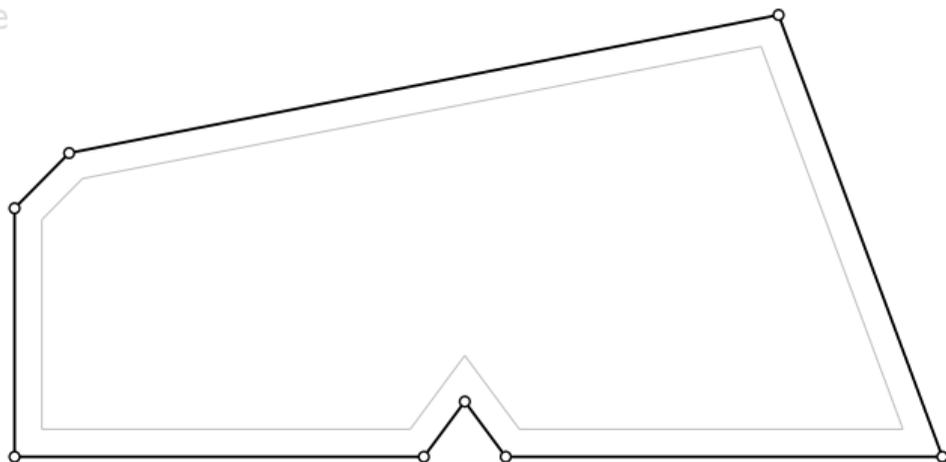
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- Defined as a result of a *wavefront propagation*.
- The *Straight Skeleton* is the trace of the vertices of the wavefront over time.
- Edge Events, Split Events.
- Applications: Tool path generation



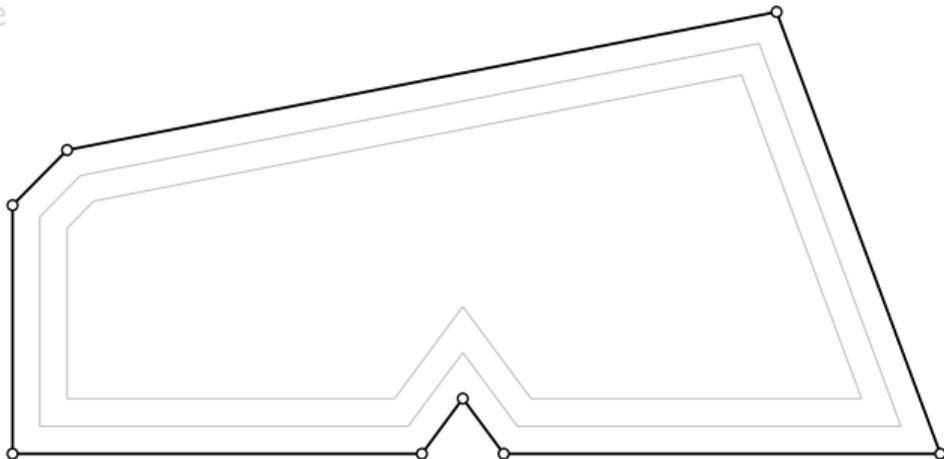
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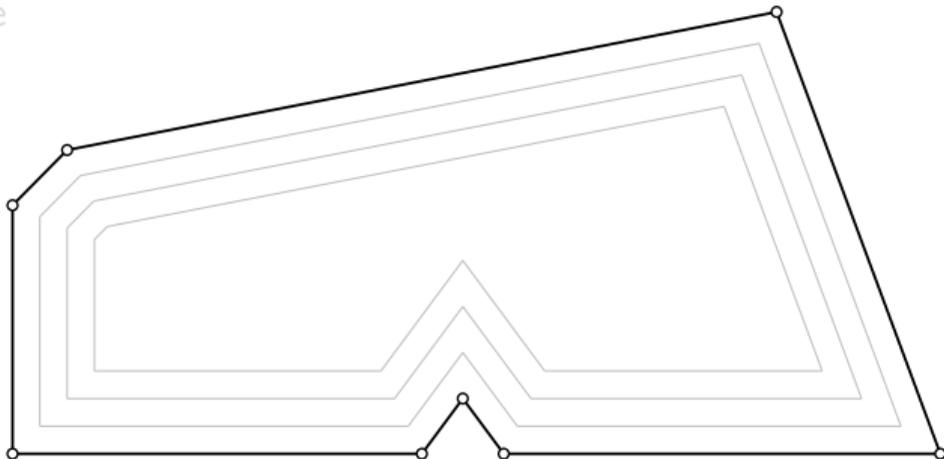
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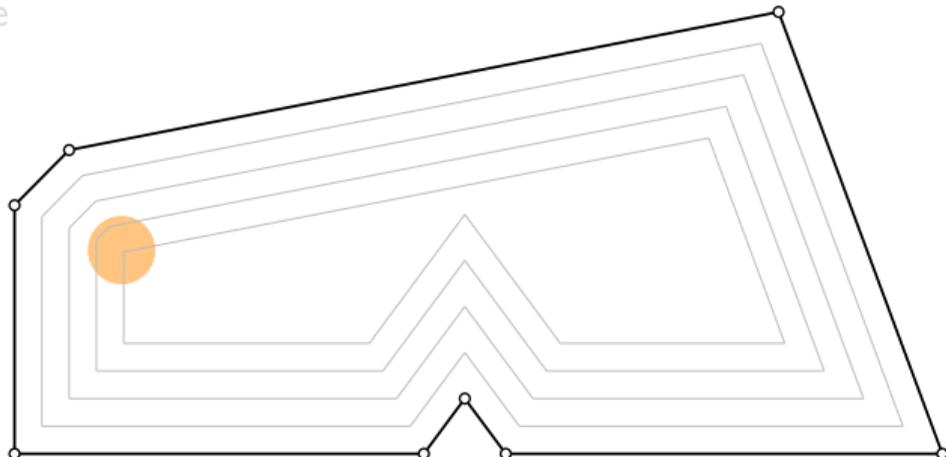
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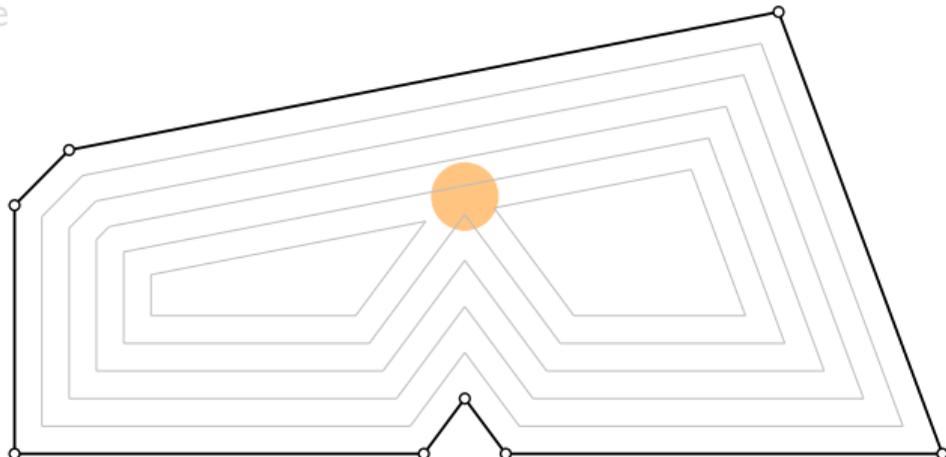
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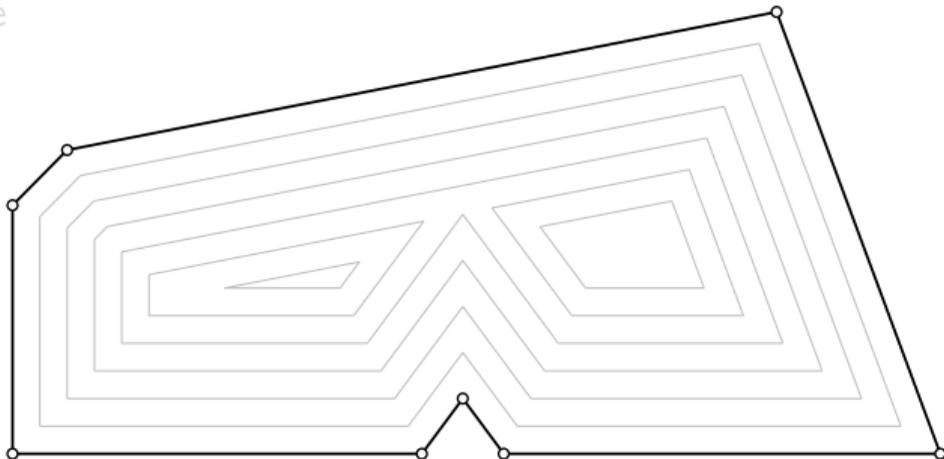
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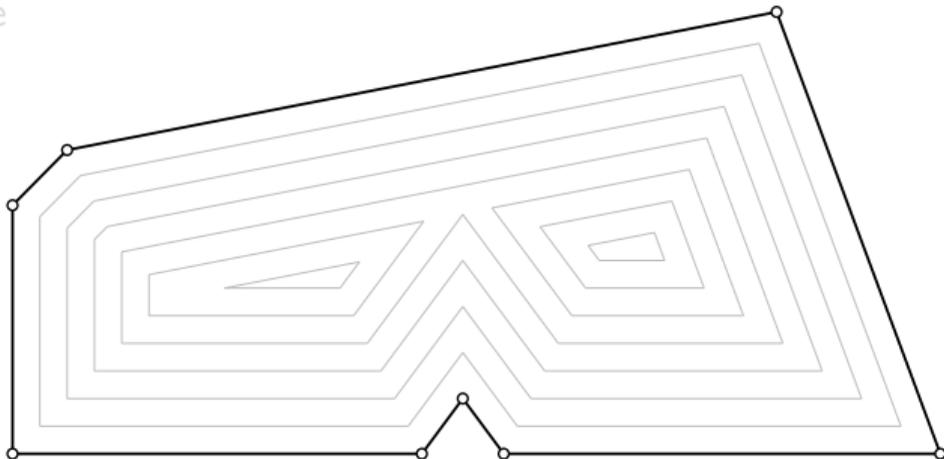
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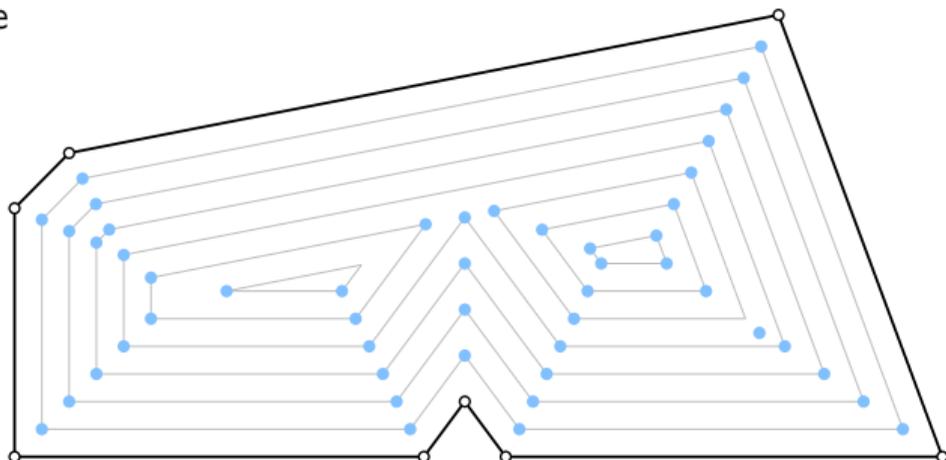
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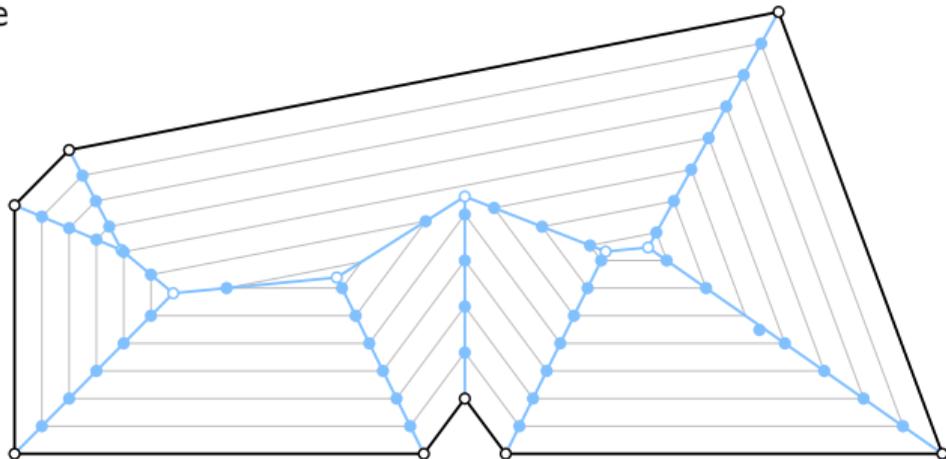
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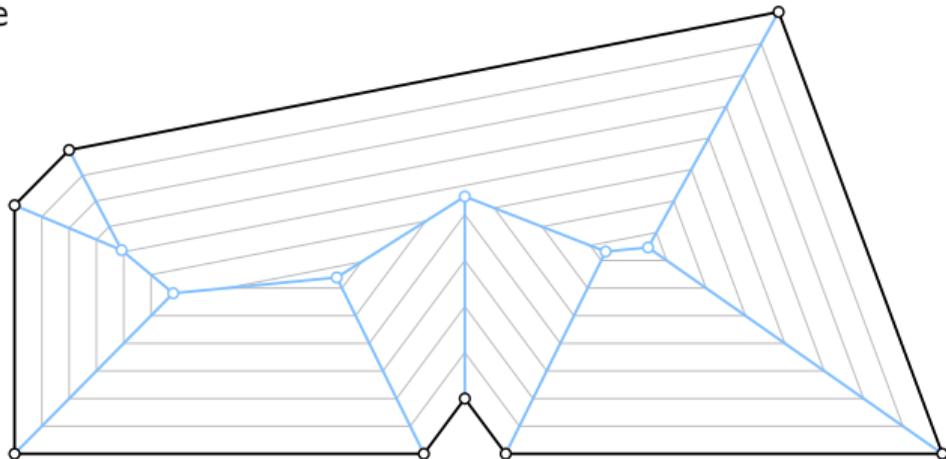
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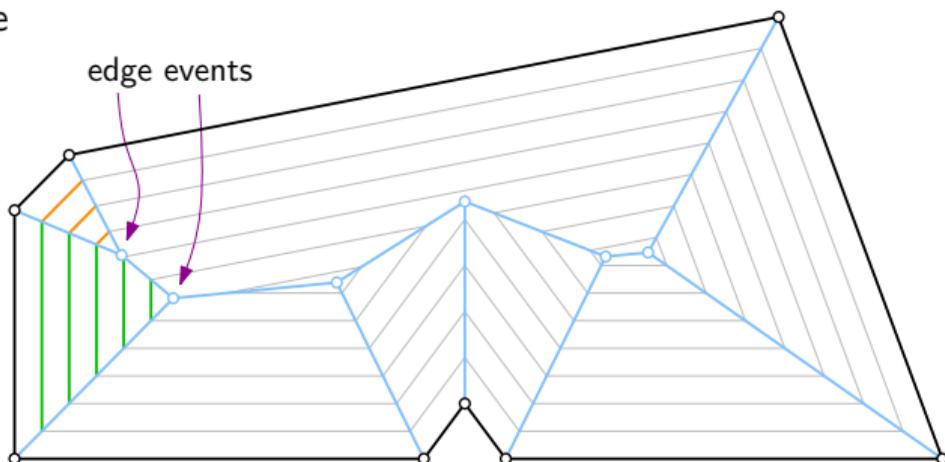
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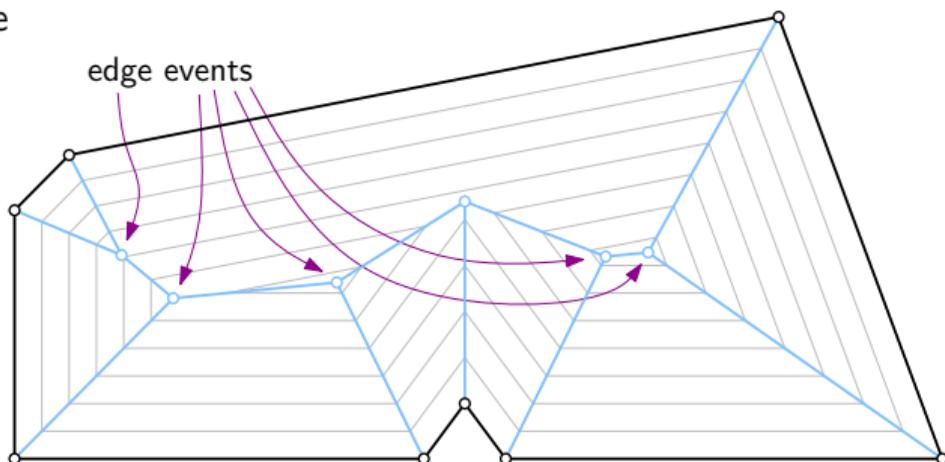
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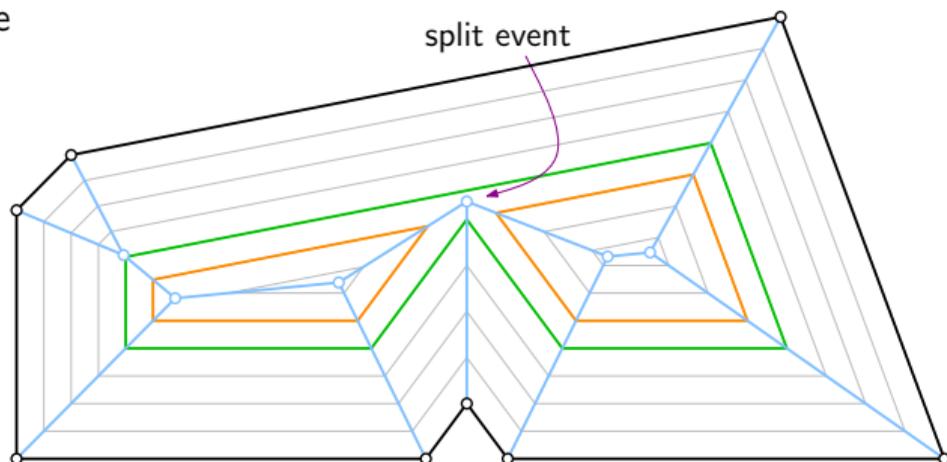
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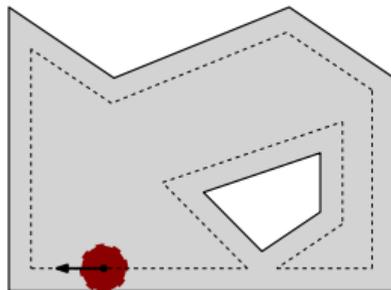
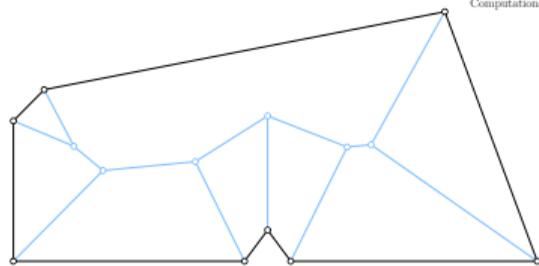
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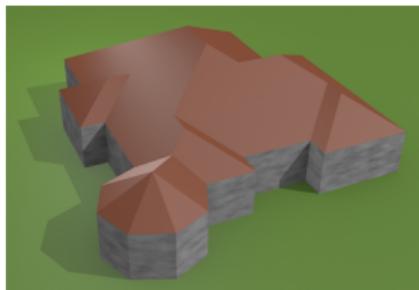
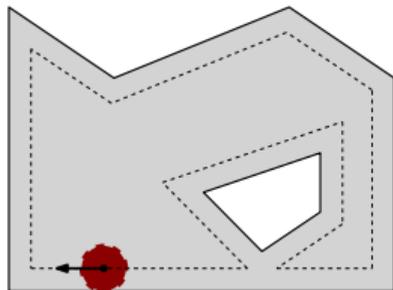
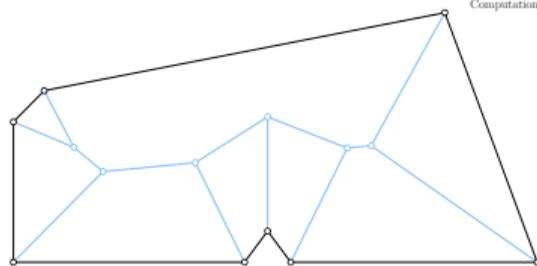
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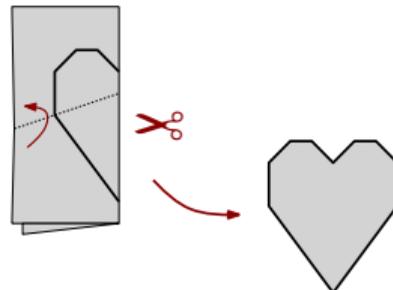
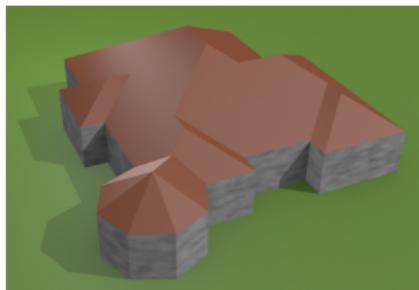
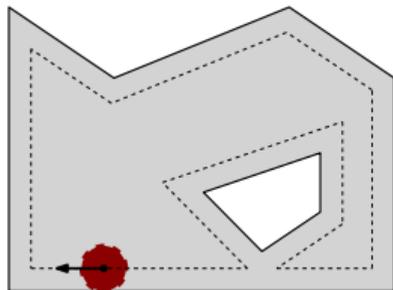
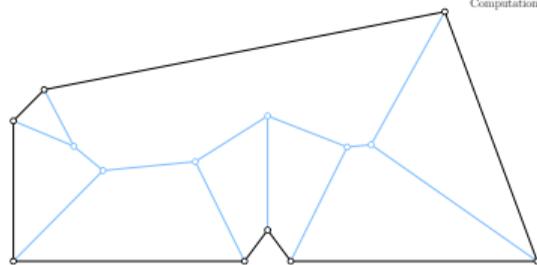
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Best worst-case complexity:

- Eppstein and Erickson (1998) and Cheng et al. (2016).

With implementations:

- Cacciola (2004), based on Felkel and Obdržálek (1998).
- Aichholzer and Aurenhammer (1998)*.
- For monotone polygons: Biedl et al. (2015)*.

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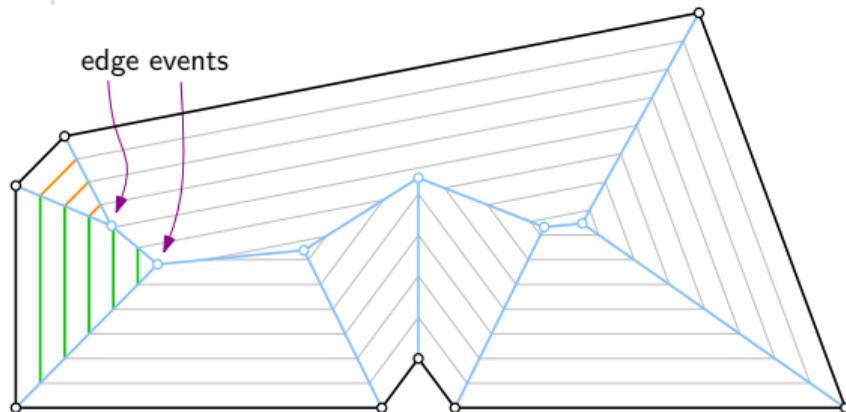
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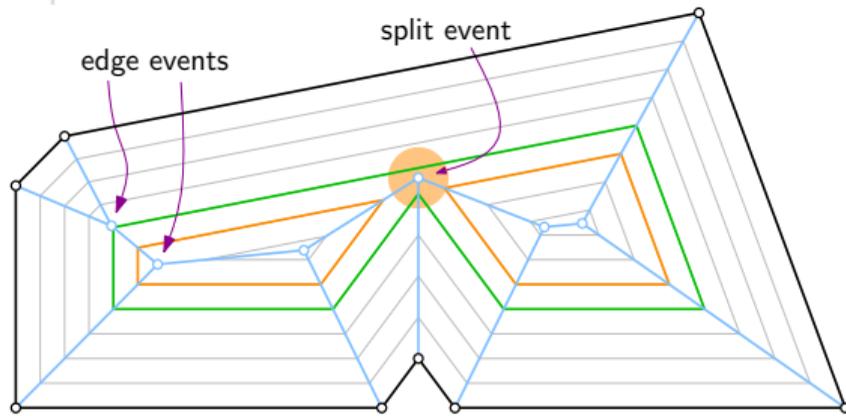
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- Input: polygons and polygons with holes.
- Priority queue of edge events and all *potential* split events.
- There are quadratic many such *potential* split events.

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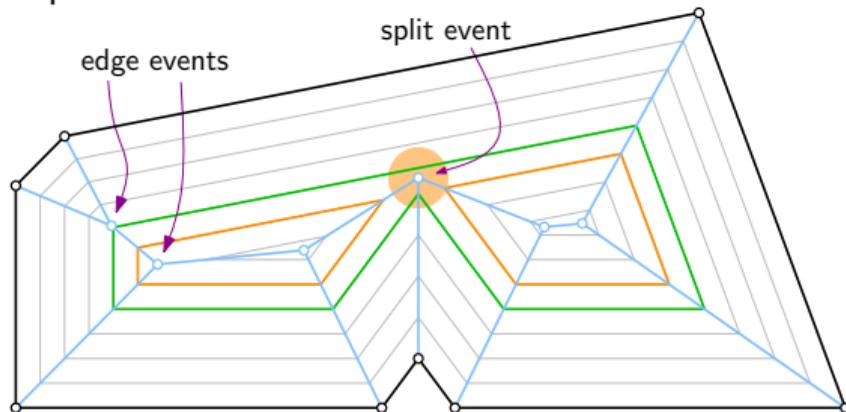
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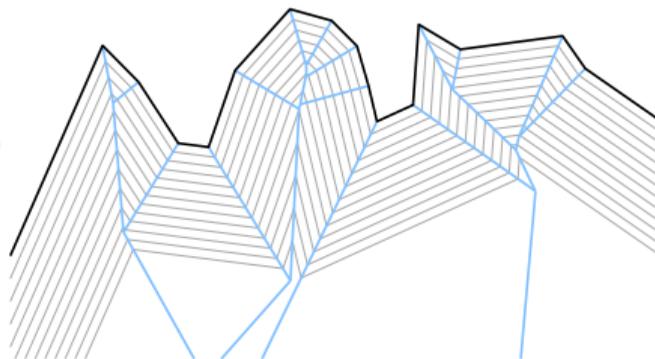


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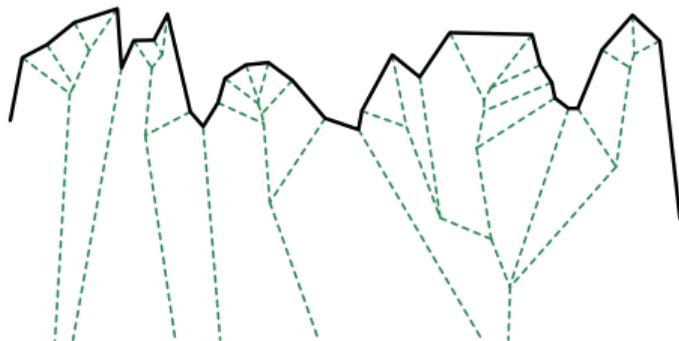
- **Input:** (strictly) monotone polygons.
 - **Key Observation:** A monotone chain never splits.
 - **Idea:** Compute the straight skeleton of two chains, then merge them.
 - **Runtime:** $\mathcal{O}(n \log n)$.
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 - Also works on *not-strictly* monotone polygons (tricky in the merge step).

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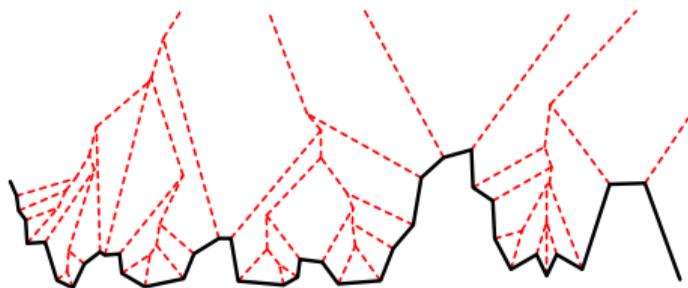
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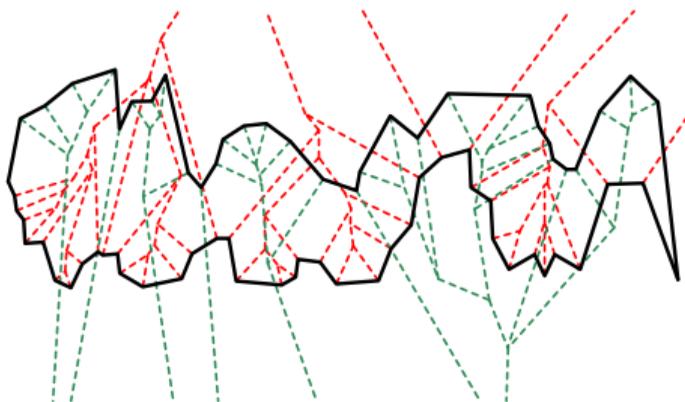
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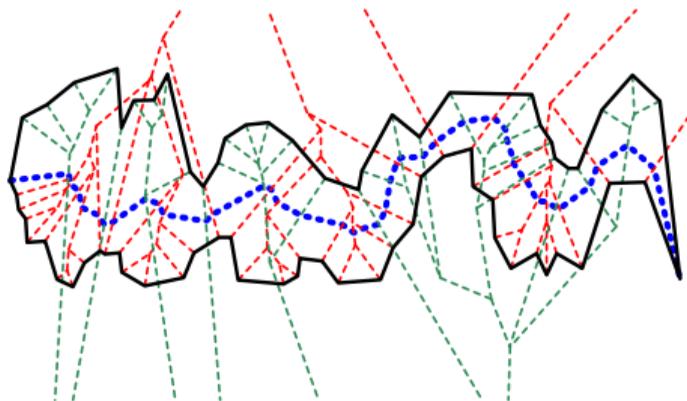
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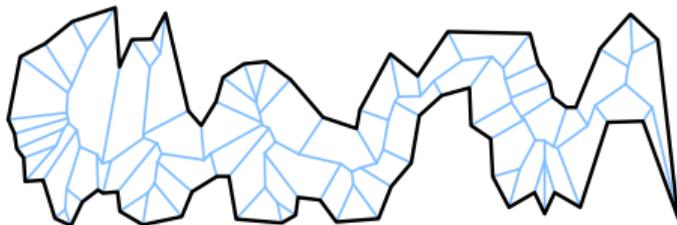
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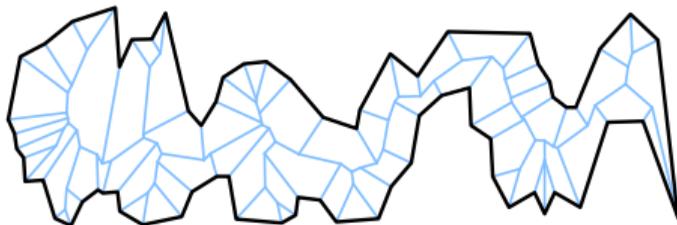
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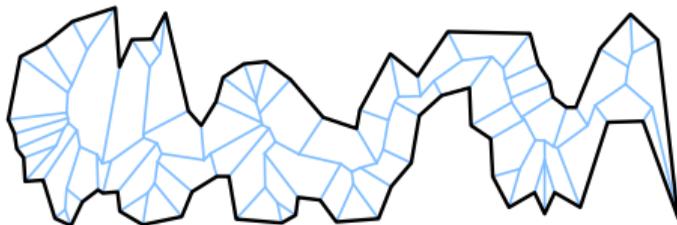
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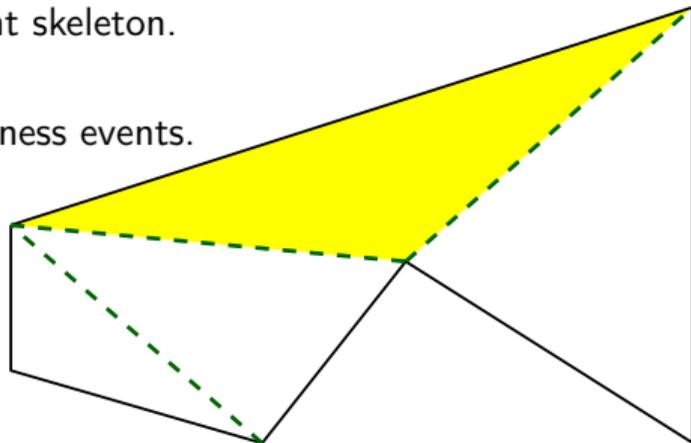
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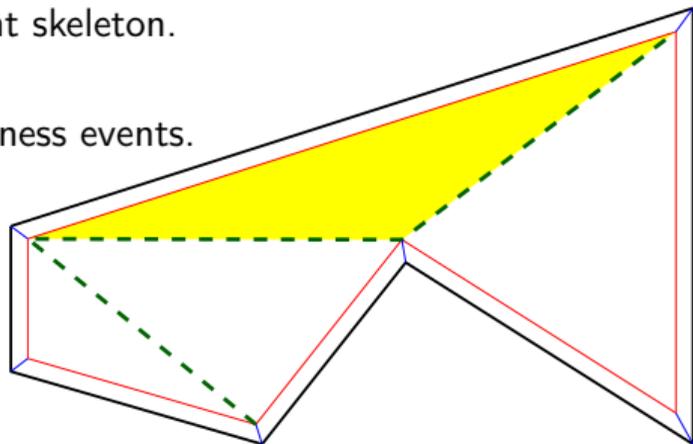
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Triangulate the not-yet-swept plane; triangles witness events.
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However, there might be $O(n^3)$ flip events.
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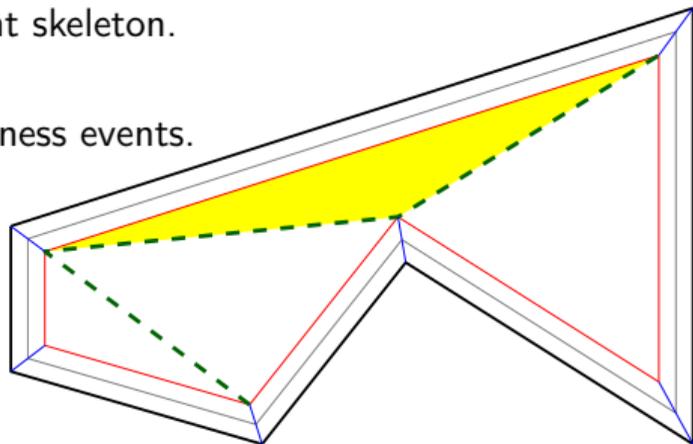
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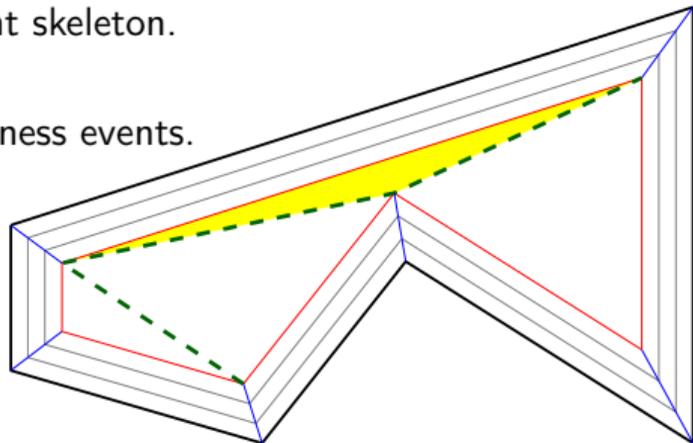


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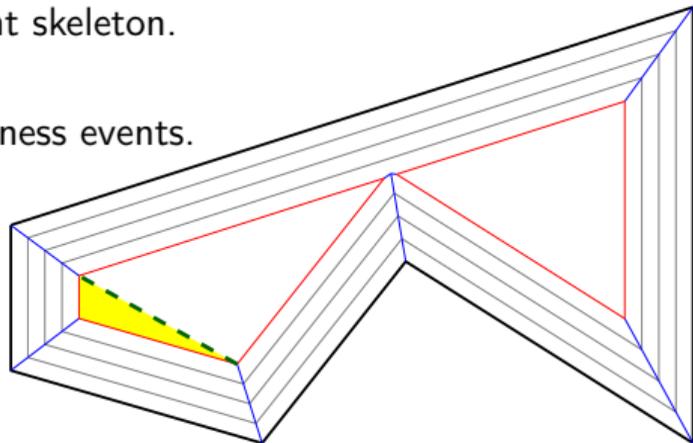


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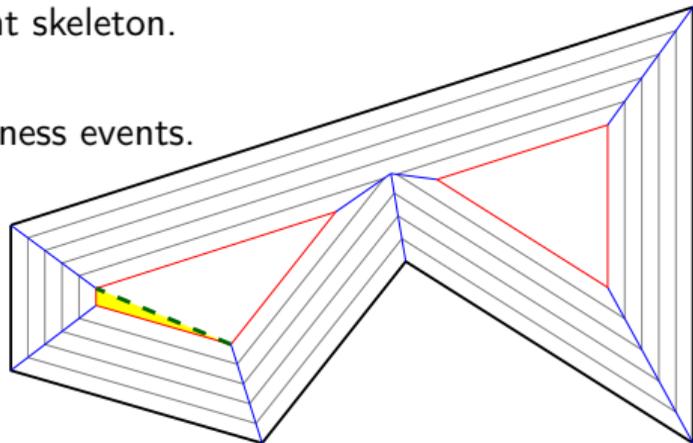
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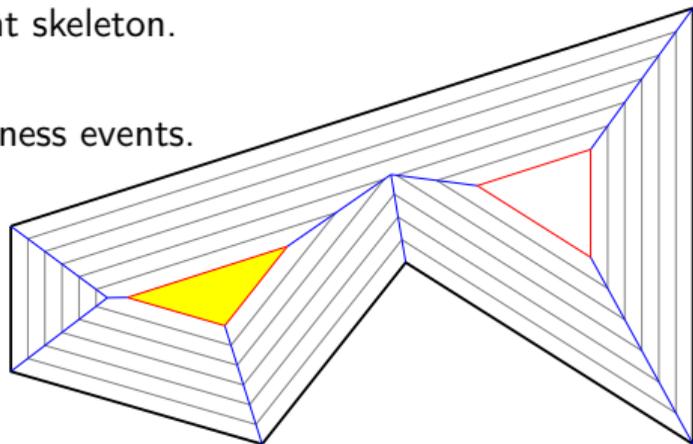
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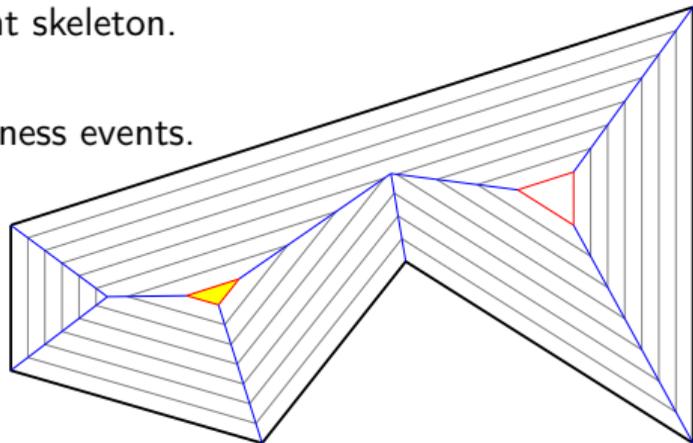
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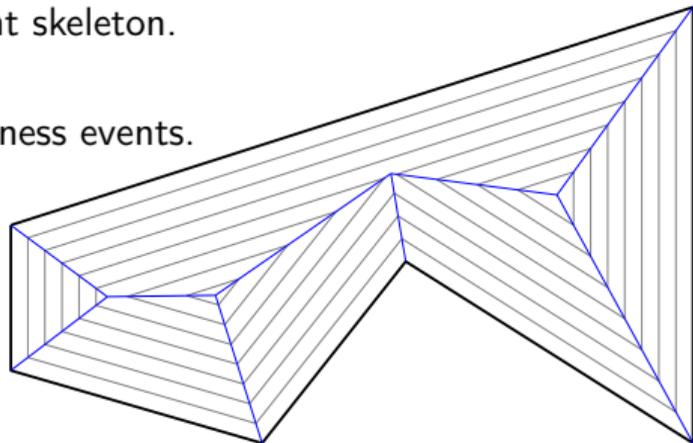
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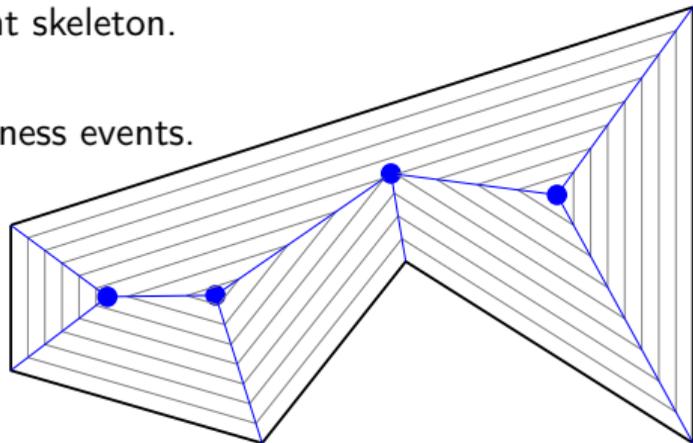
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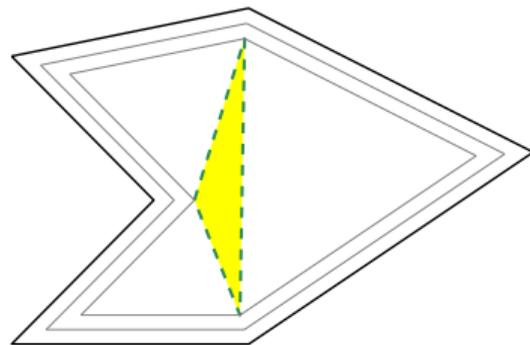
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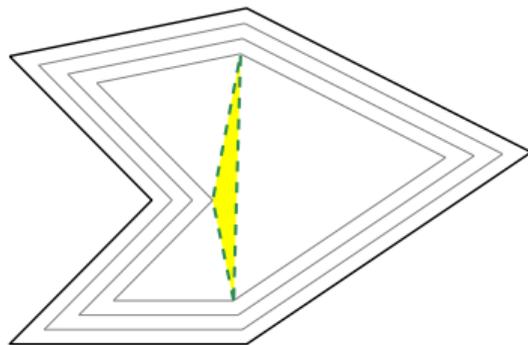
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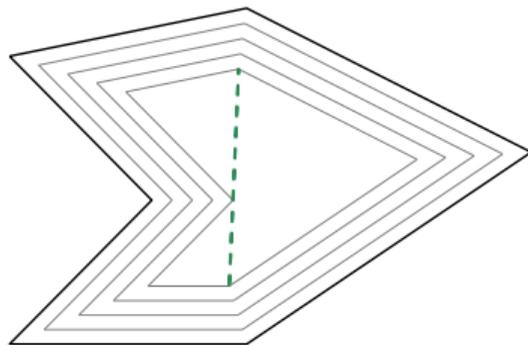
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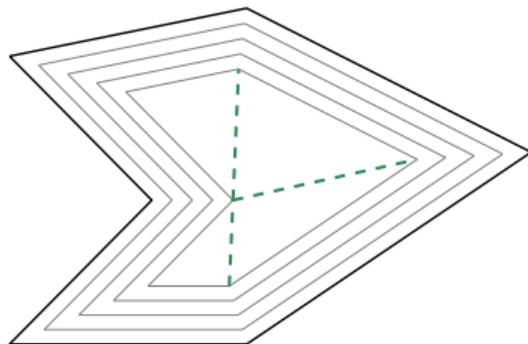
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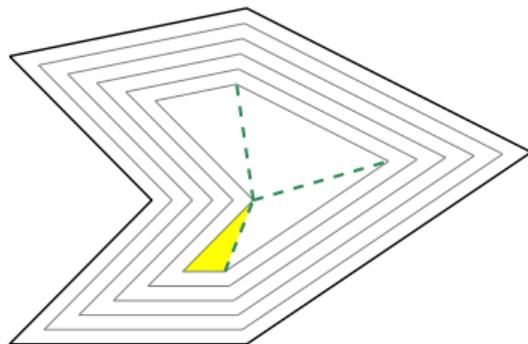
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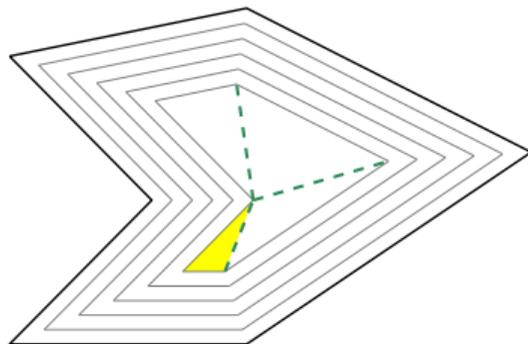
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- Several special cases not considered in the original paper.

- Input: PSLGs. Can compute the weighted straight skeleton.
- Uses a kinetic data structure to witness events:
Triangulate the not-yet-swept plane; triangles witness events.
- There are only linear many *real events*.
However, there might be $O(n^3)$ flip events.



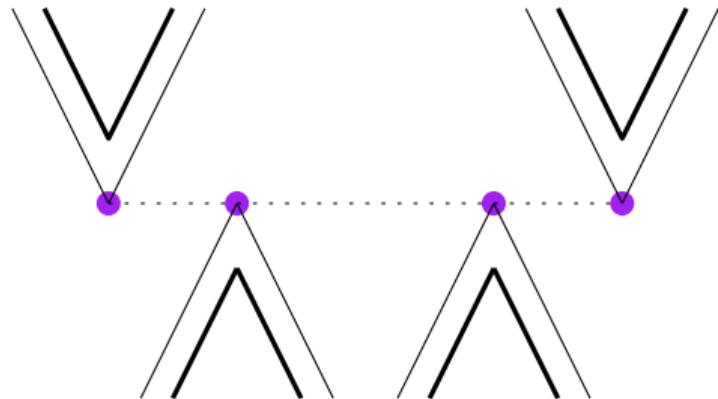
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Some Special Cases

- Flip-event Loops.
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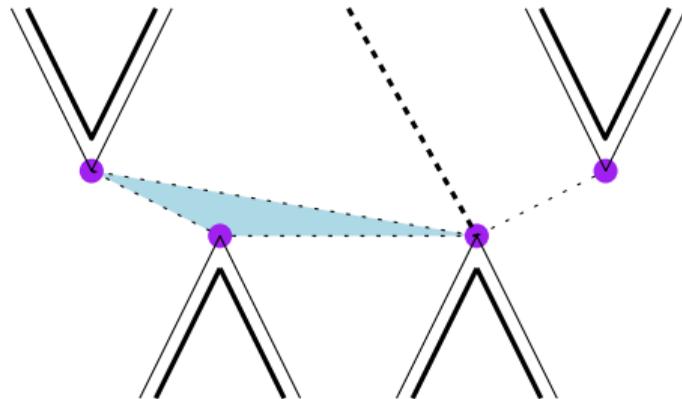


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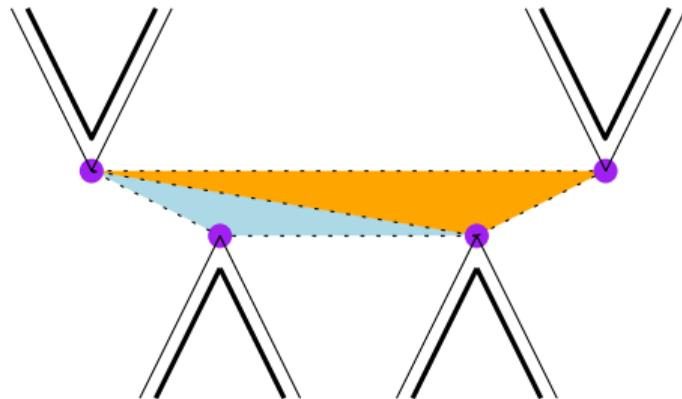


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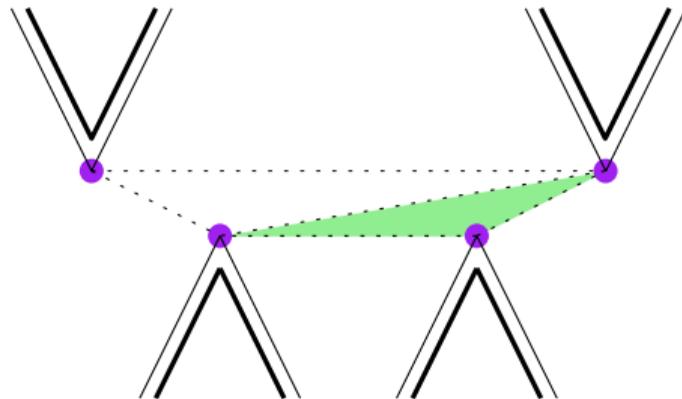


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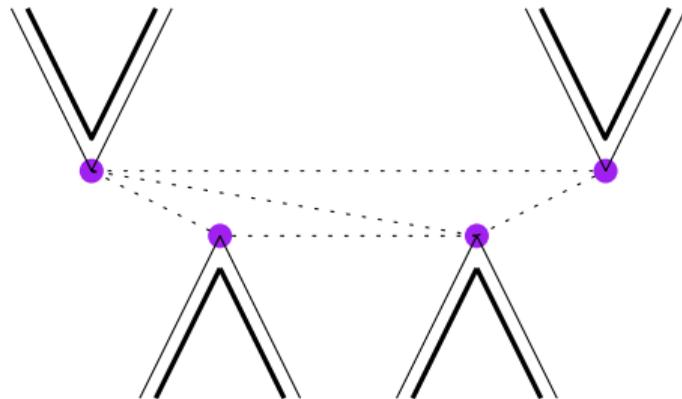


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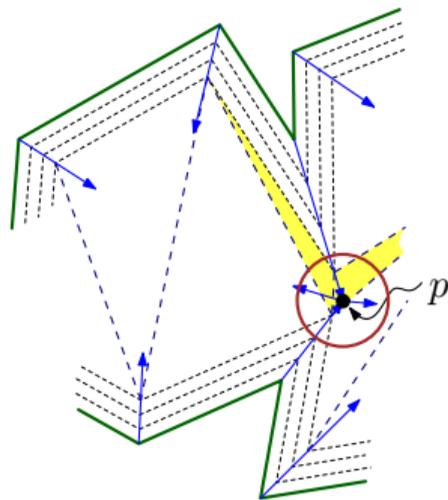


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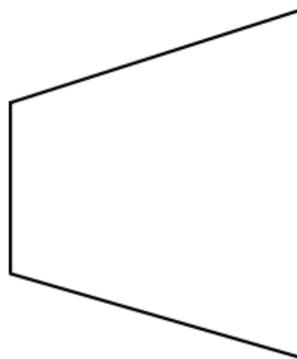


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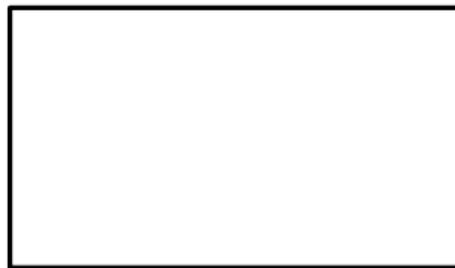
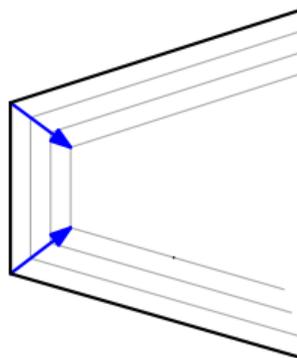


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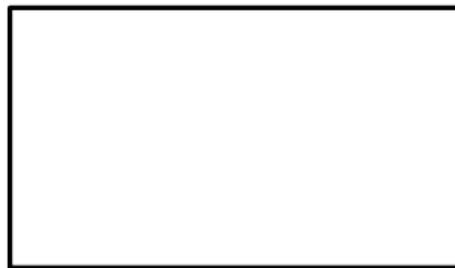
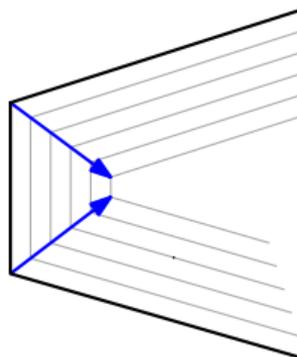


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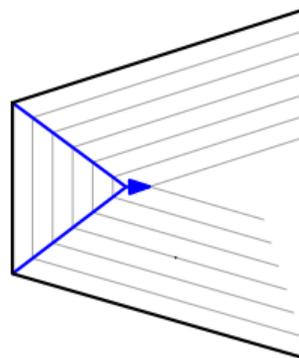


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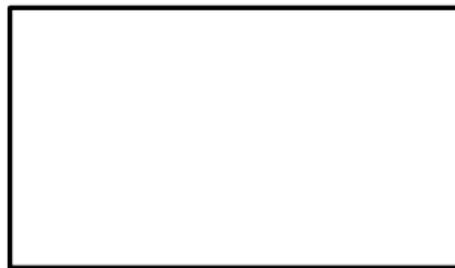
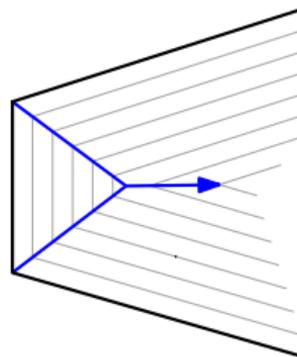


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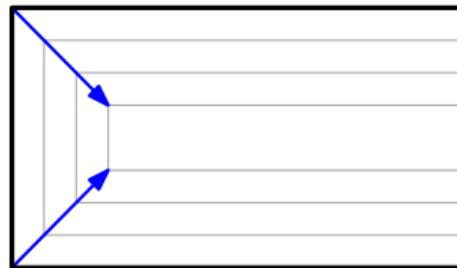
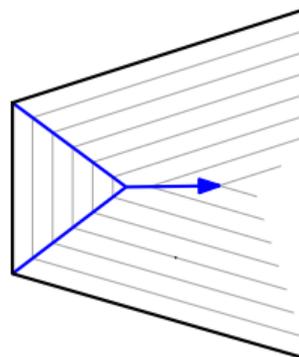


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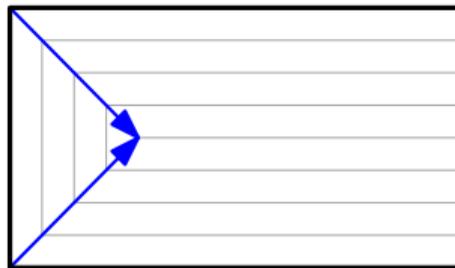
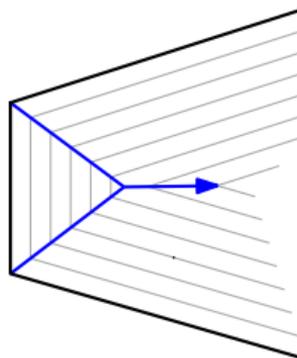


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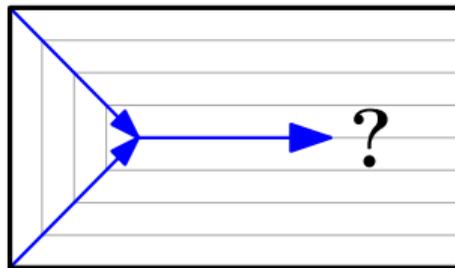
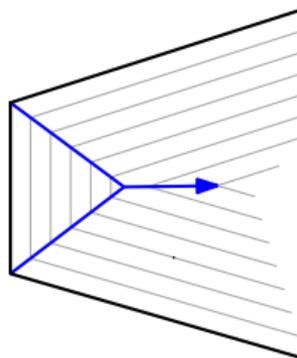


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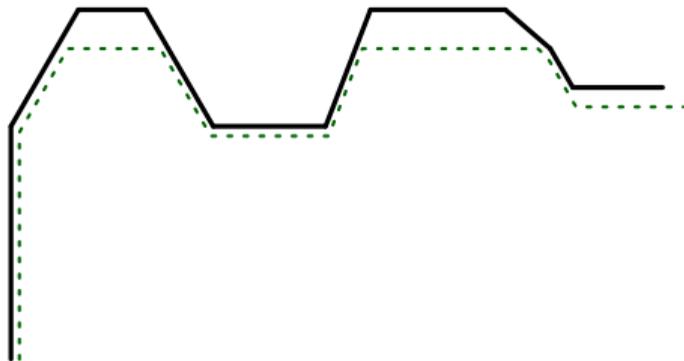


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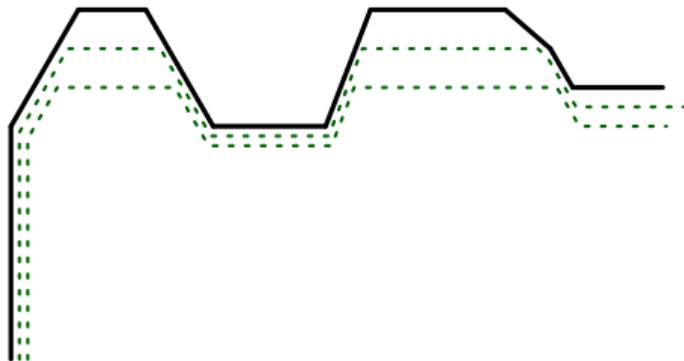


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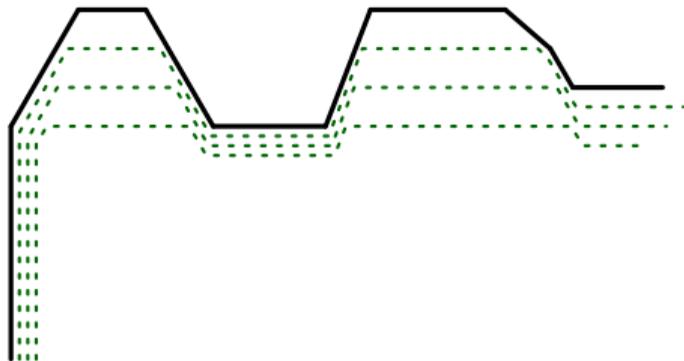


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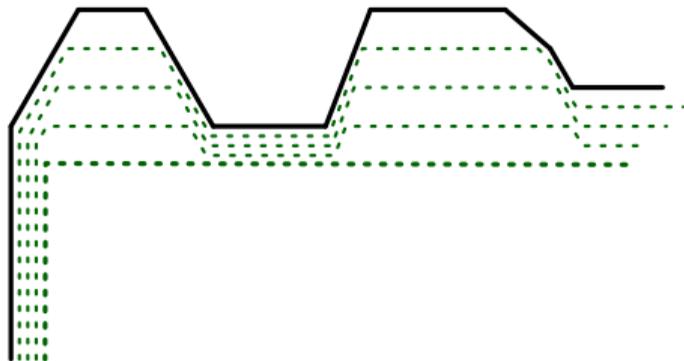


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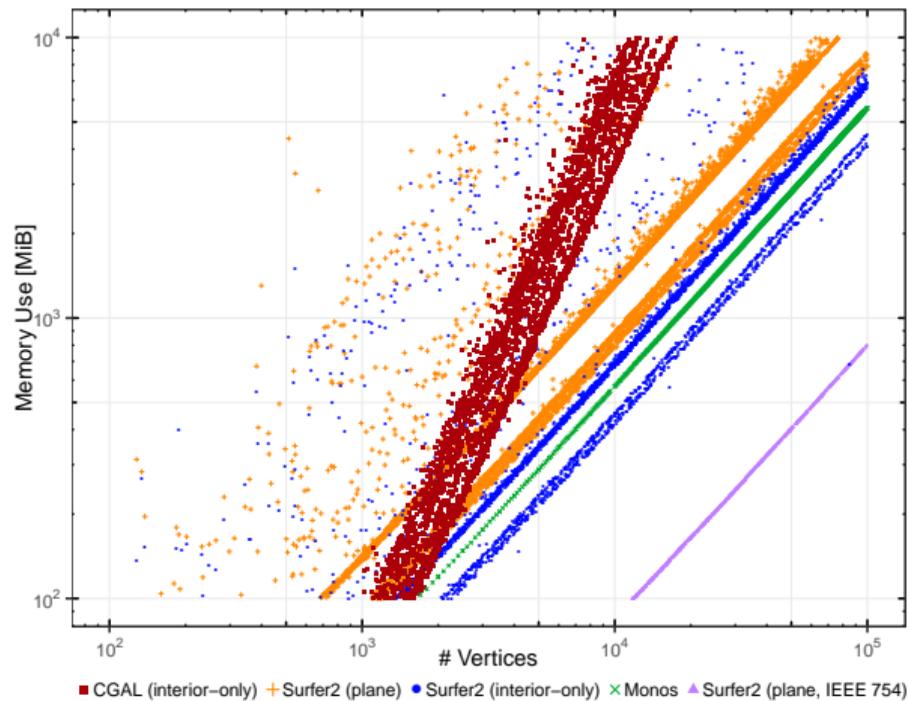
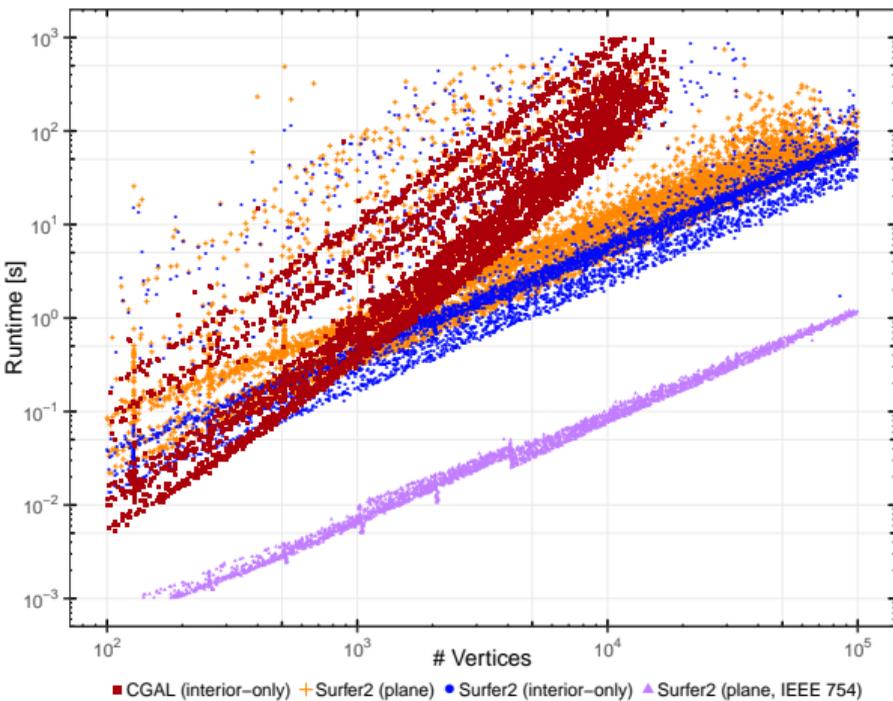
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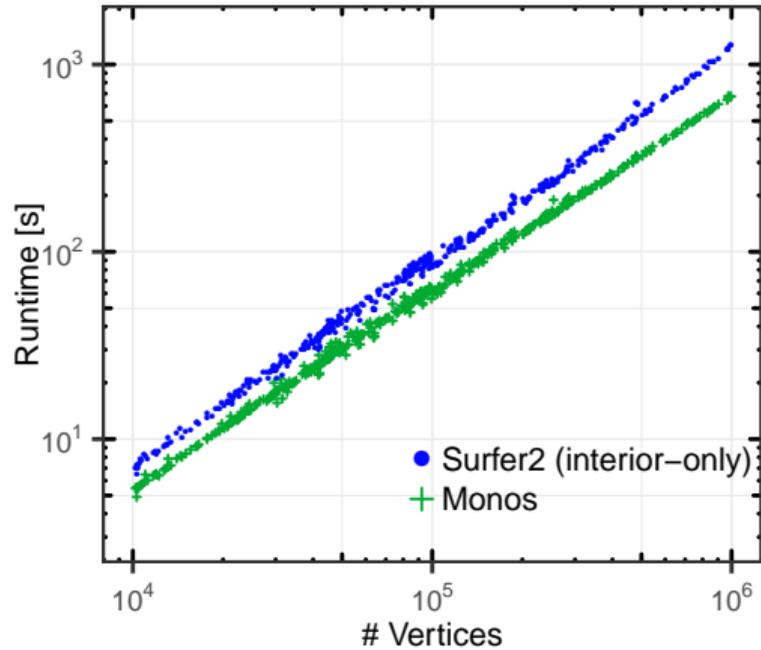
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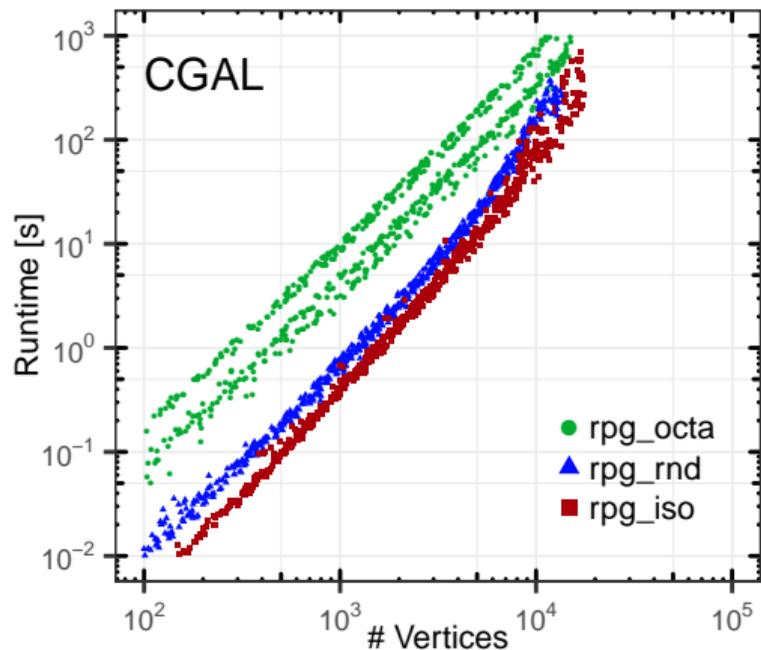
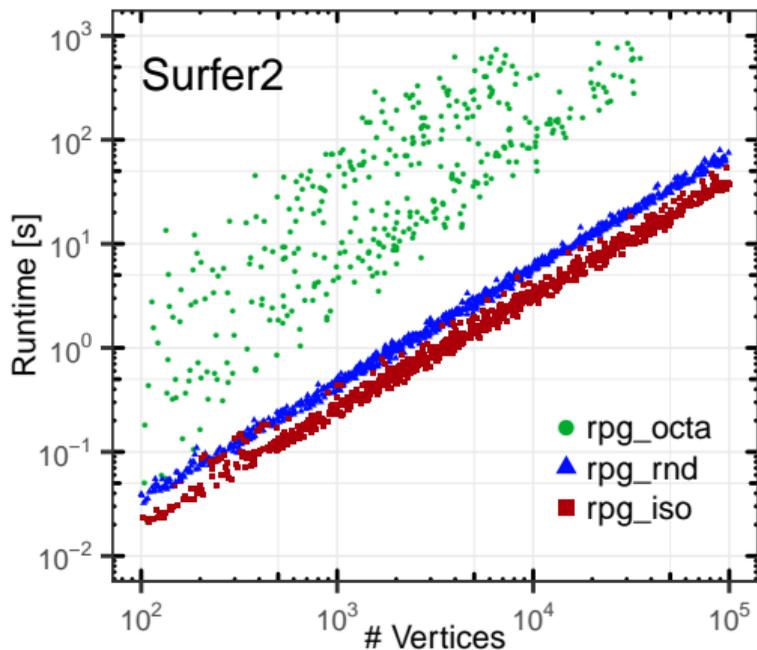
CGAL v. SURFER2



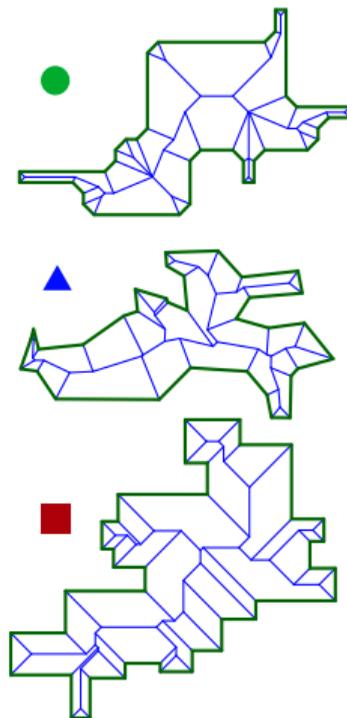
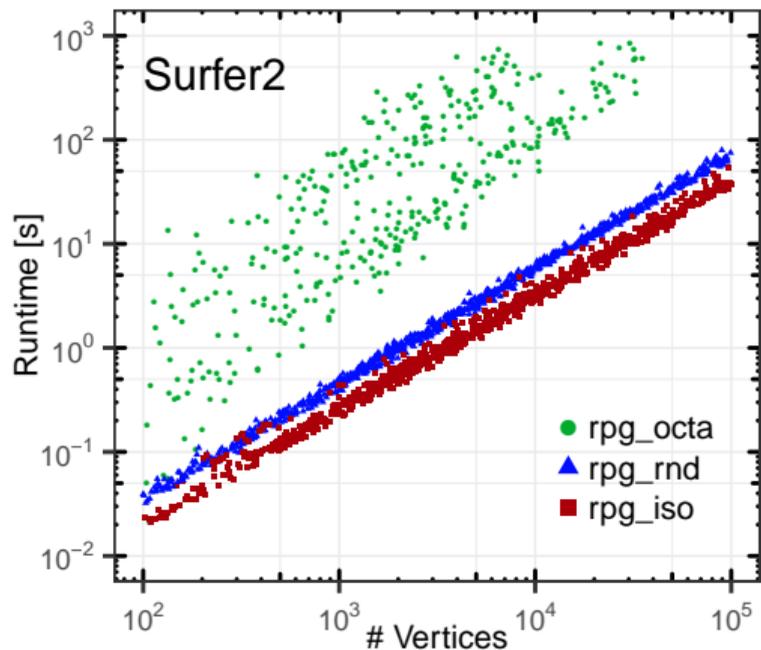
MONOS v. SURFER2



Investigating the spread

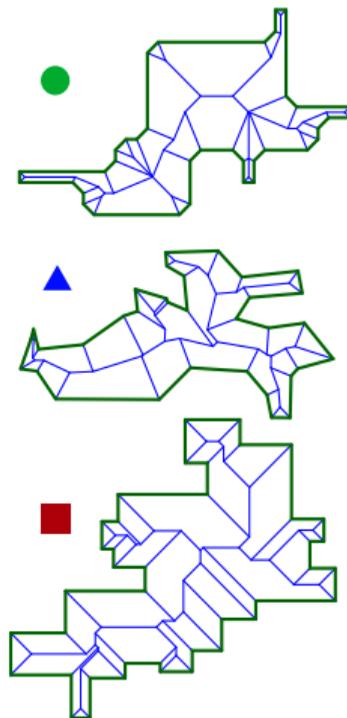
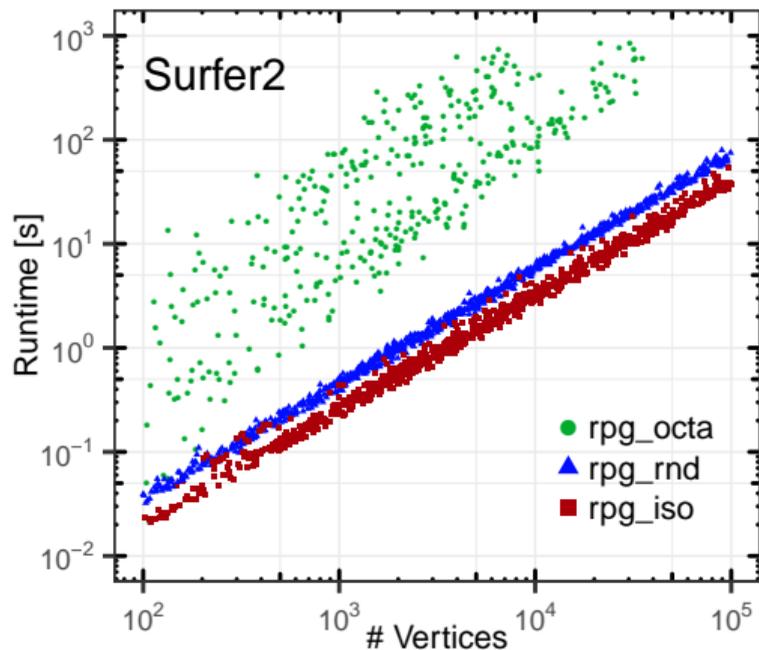


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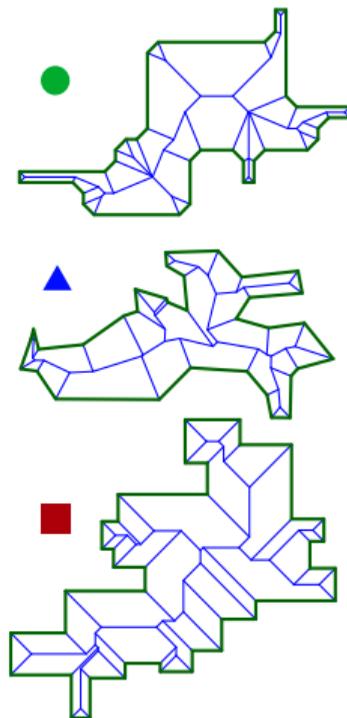
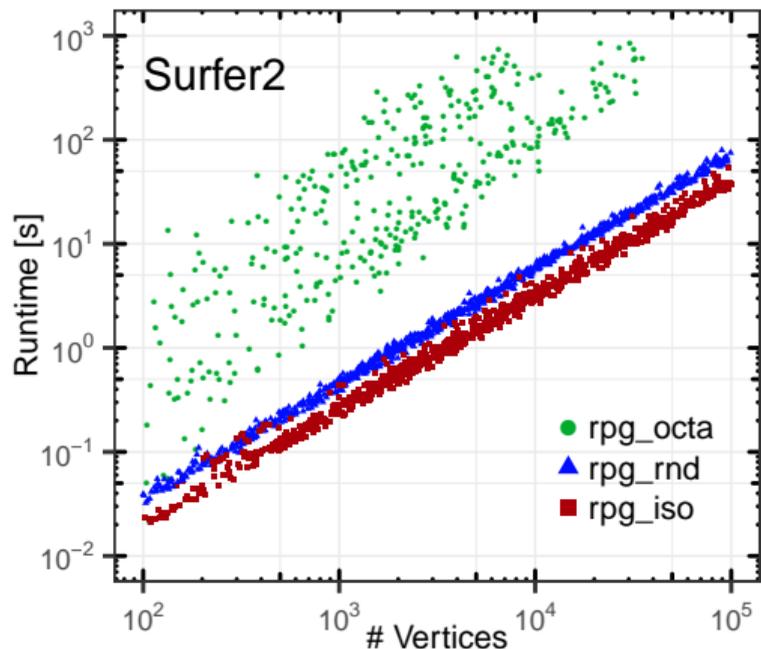
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- Turns out our octa input was on the integer grid, the iso had random coordinates.
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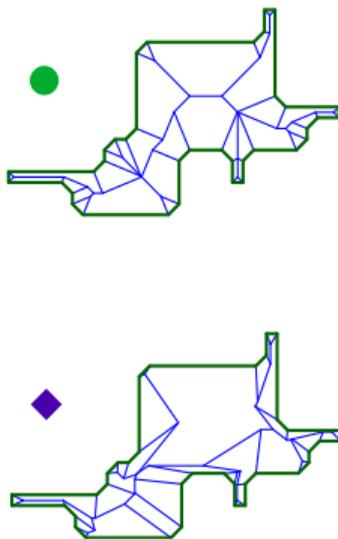
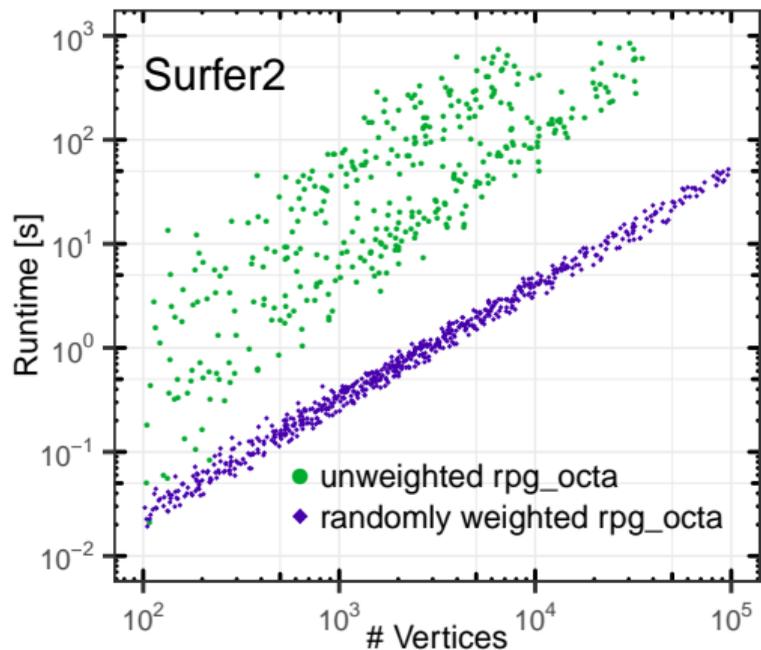
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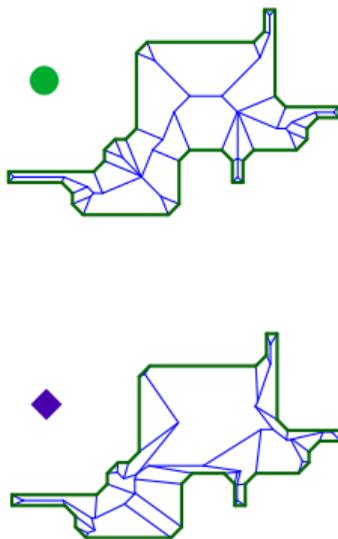
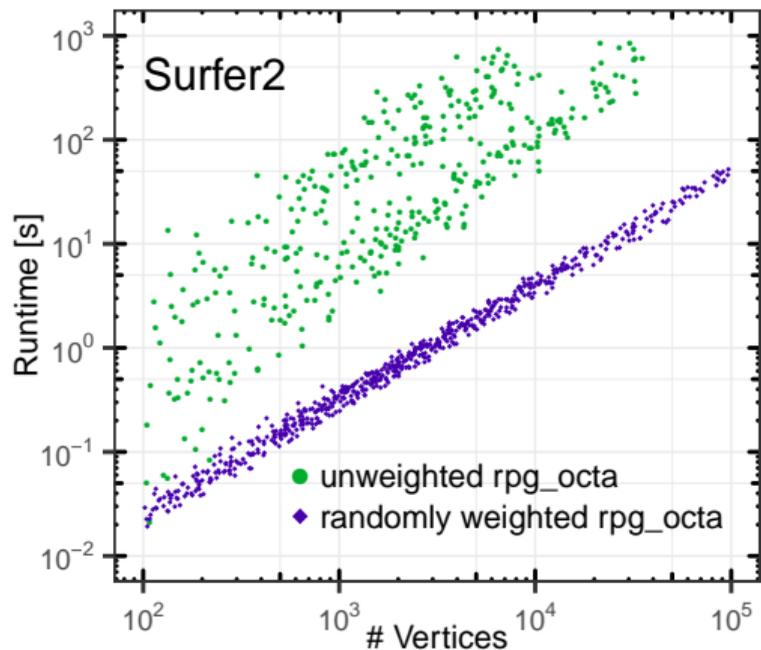
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Source code

- MONOS: <https://github.com/cgalab/monos>
- SURFER2: <https://github.com/cgalab/surfer2>

Thanks!

Questions? Mail palfrader@cs.sbg.ac.at

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