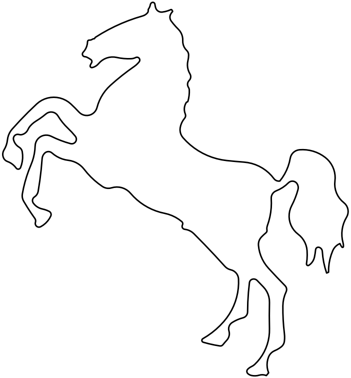
COMPUTING STRAIGHT SKELETONS BY MEANS OF KINETIC TRIANGULATIONS

Peter Palfrader

October 2013

- Aichholzer, Alberts, Aurenhammer, Gärtner 1995.
- Problem: Given input graph, find the *straight skeleton*.



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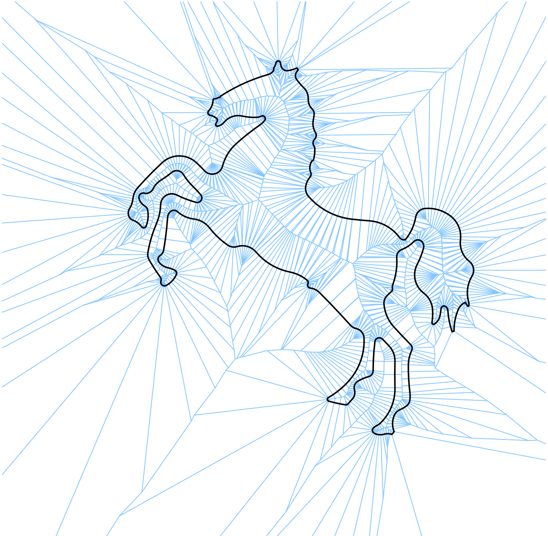
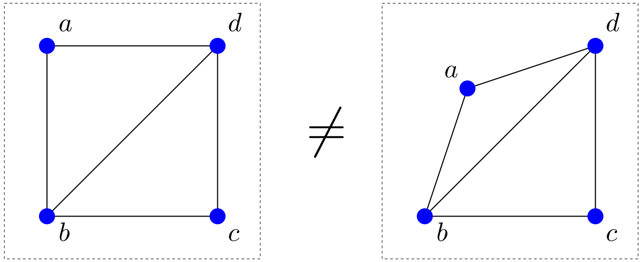
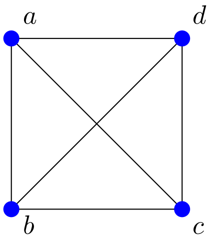


- When we consider a graph $G = (V, E)$ we always consider its *embedding*.



PRELIMINARIES

- When we consider a graph $G = (V, E)$ we always consider its *embedding*.
- We care about planar, straight-line graphs or PSLGs.



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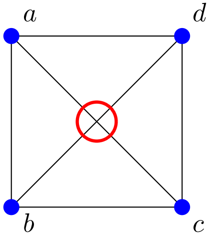

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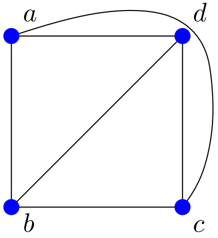

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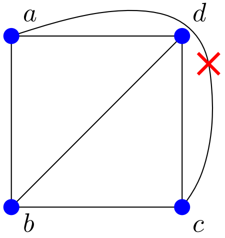

Straight Skeletons – Motivation

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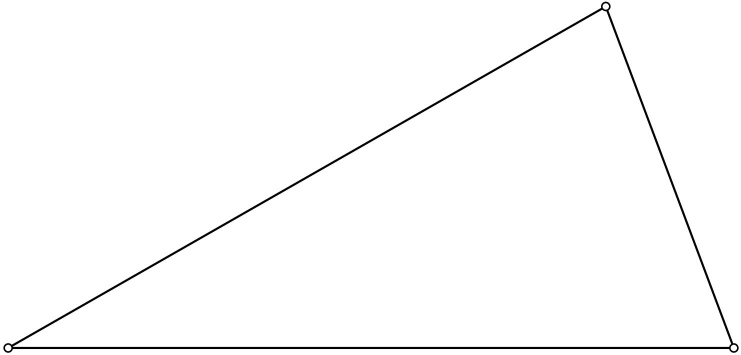
- input polygon $\mathcal{P}$ emanates wavefront $\mathcal{WF}(\mathcal{P}, t)$.



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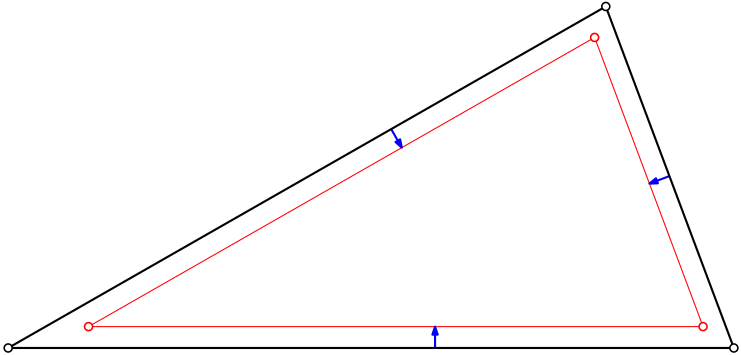

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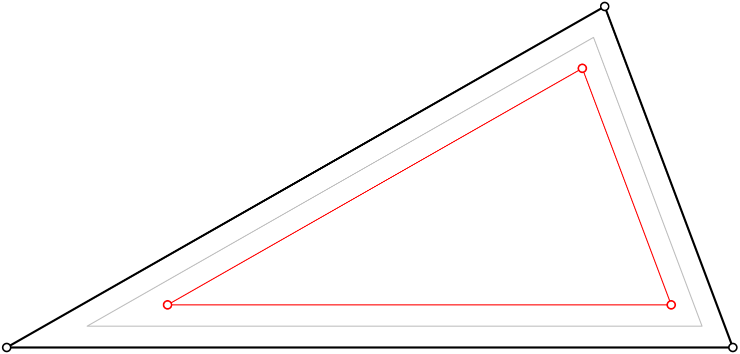

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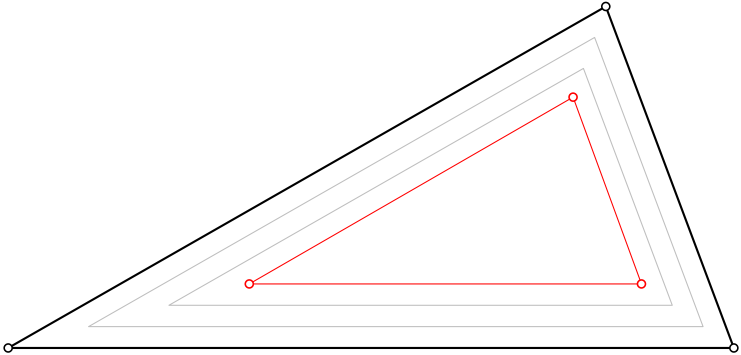

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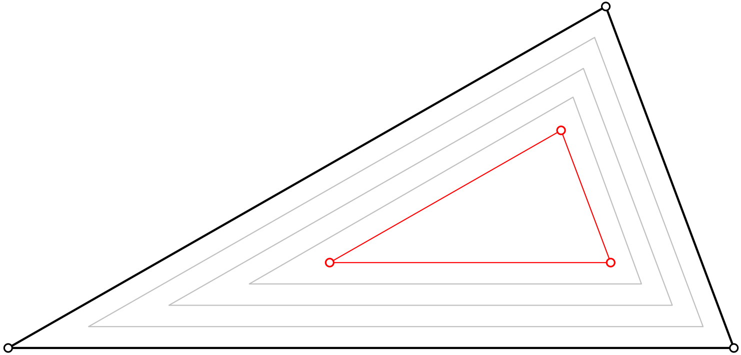

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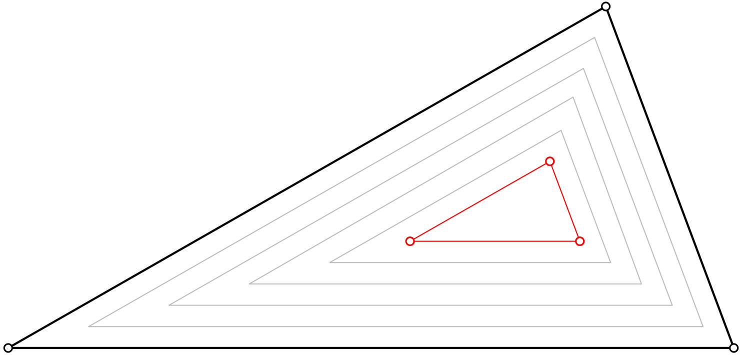

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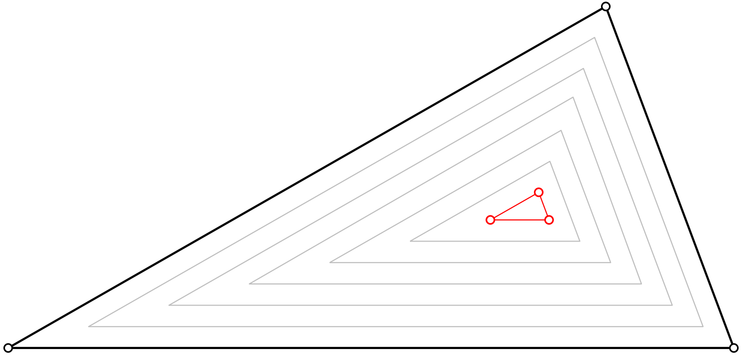

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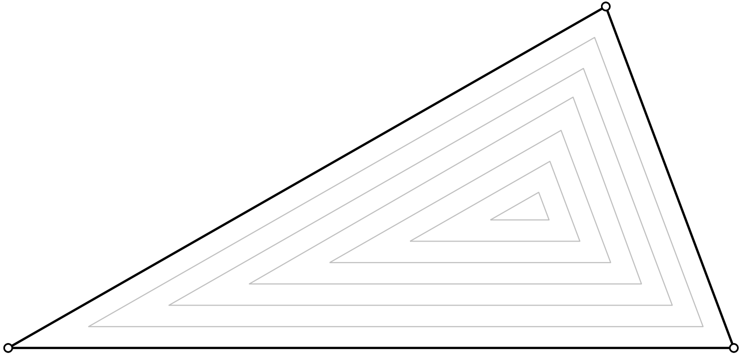

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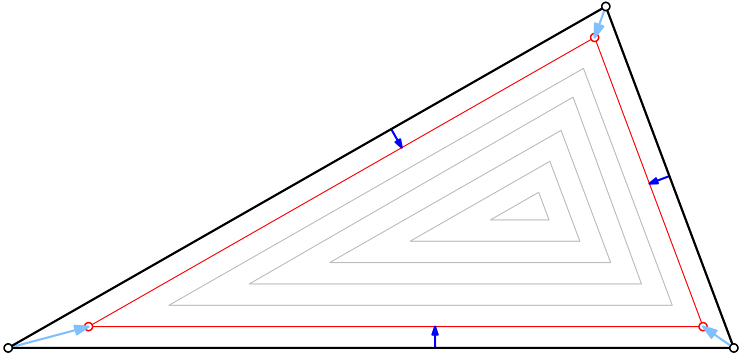

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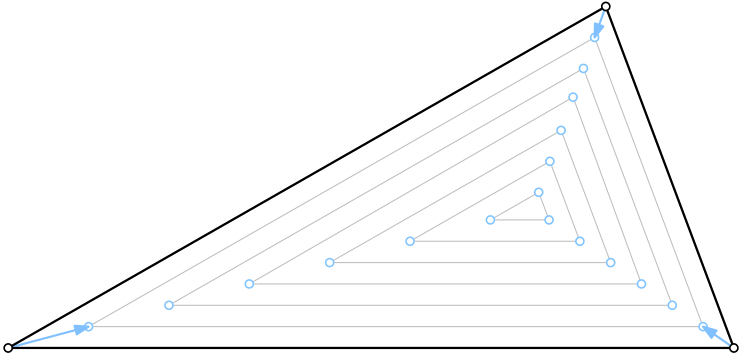

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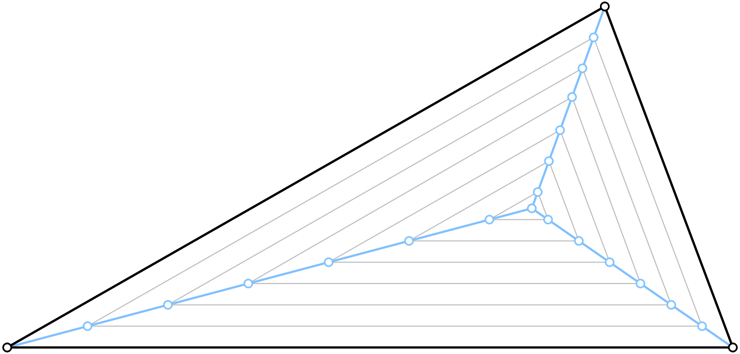

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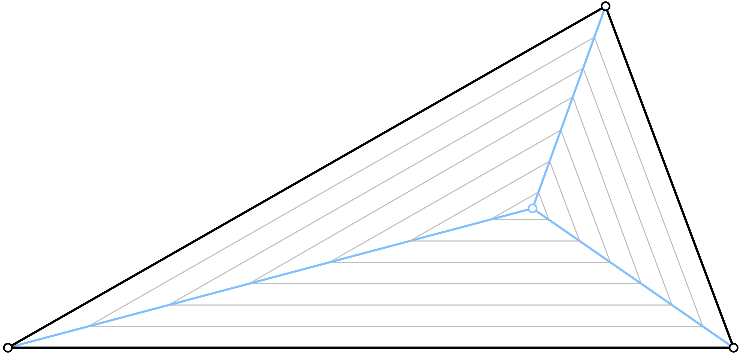

TOPOLGY CHANGES – EDGE EVENTS

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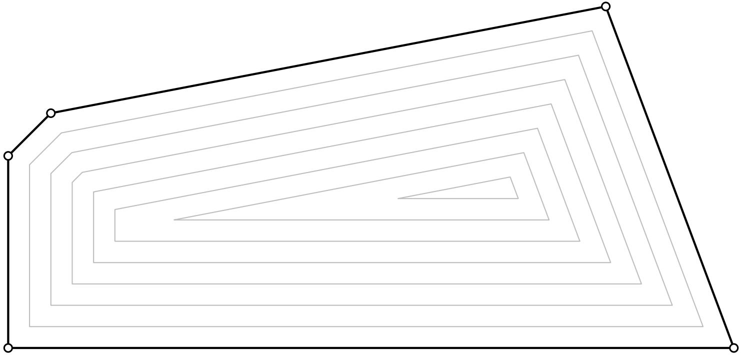

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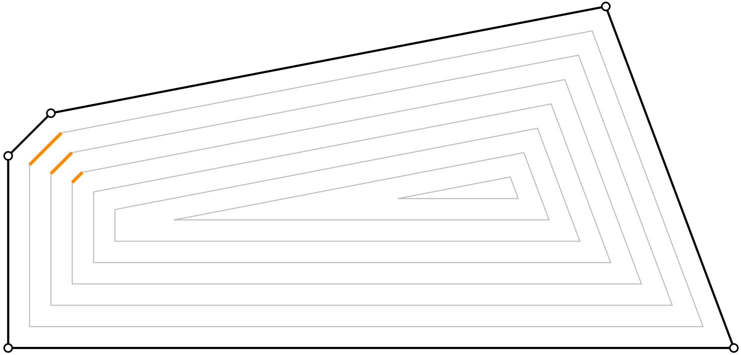

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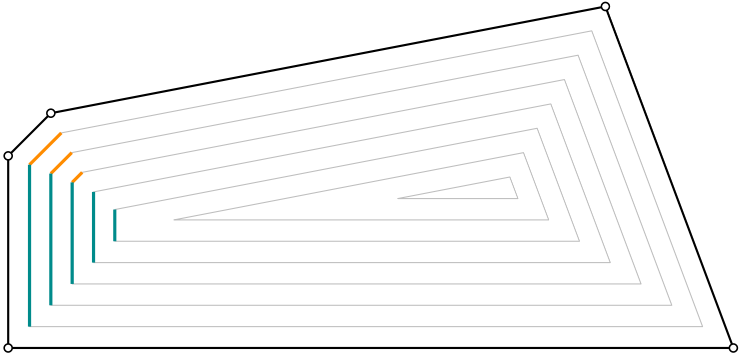

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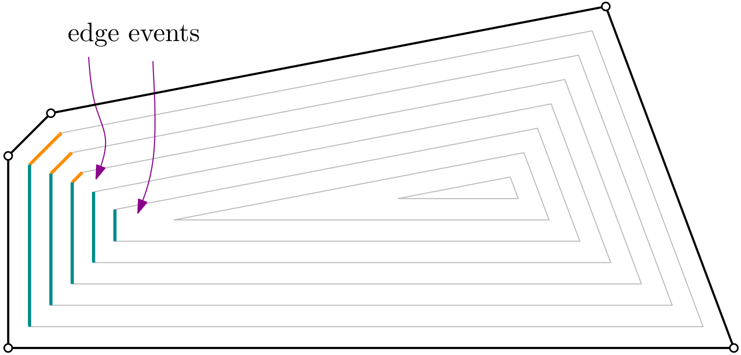

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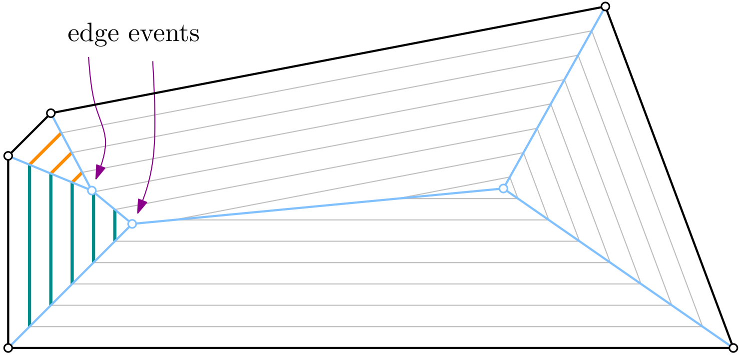

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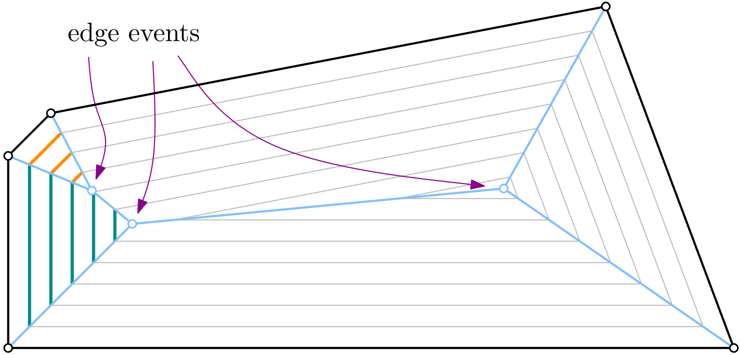

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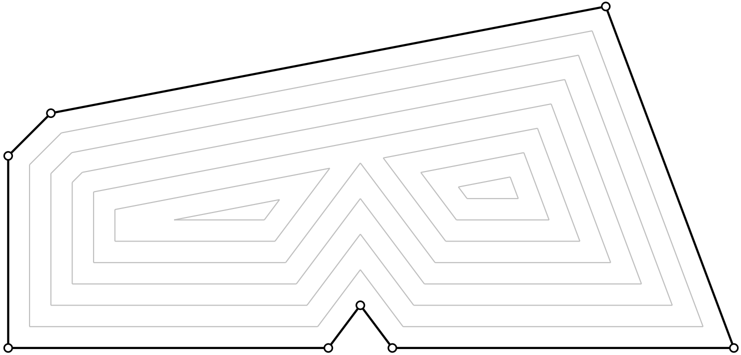

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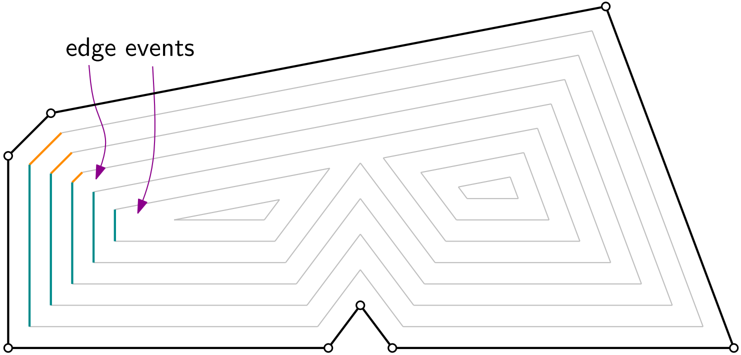

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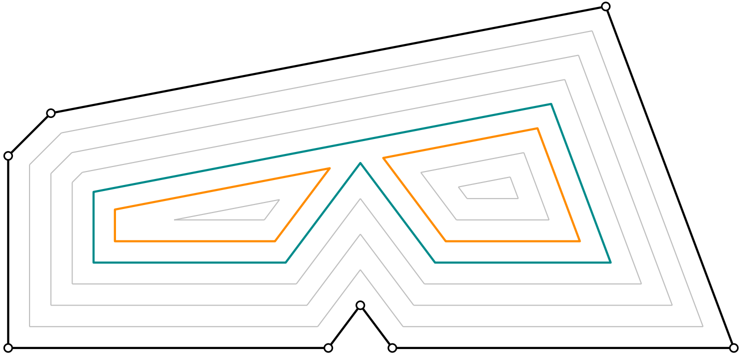

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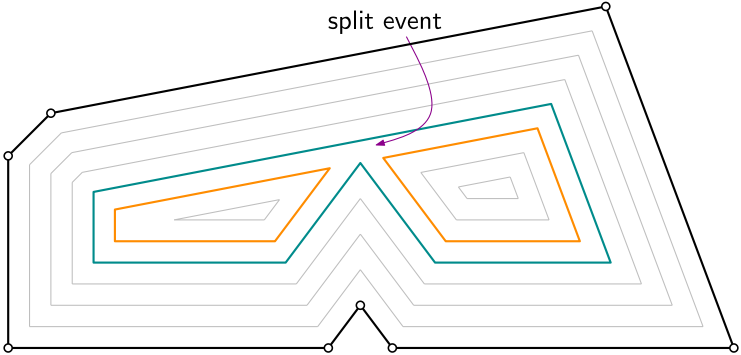

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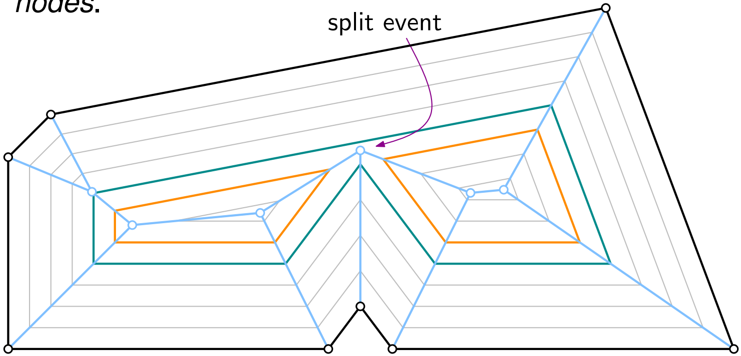

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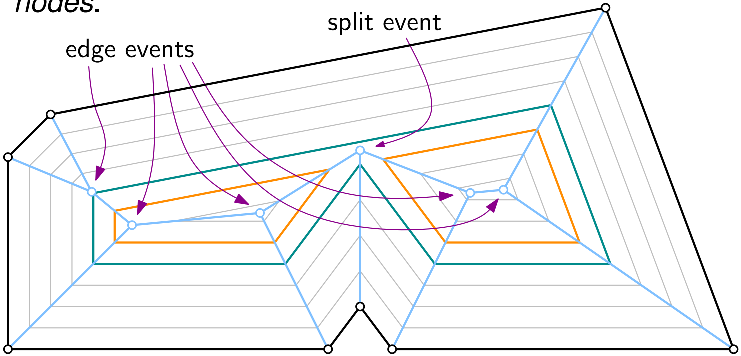
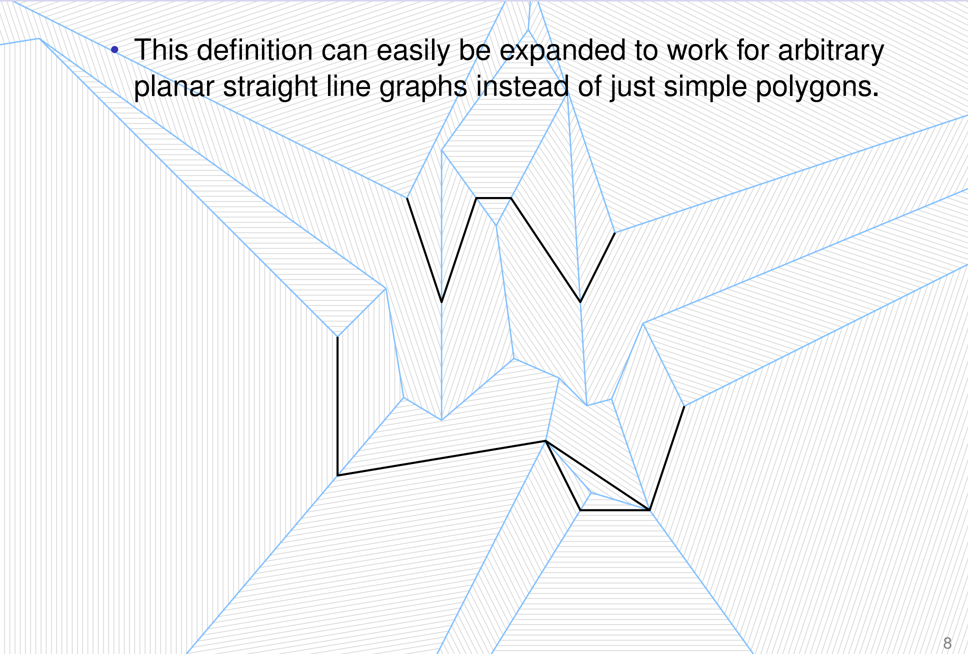


Straight Skeleton of a PSLG

- This definition can easily be expanded to work for arbitrary planar straight line graphs instead of just simple polygons.



Summary: Straight Skeletons

- The straight skeleton is the union of traces of wavefront vertices over the propagation process.
- The topology of the wavefront changes with time due to edge and split events. These are witnessed in $\mathcal{SK}$ as nodes.

Applications: Roof Modeling

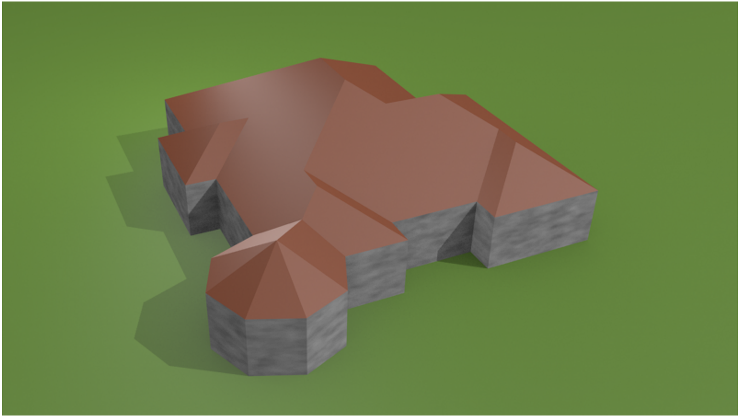
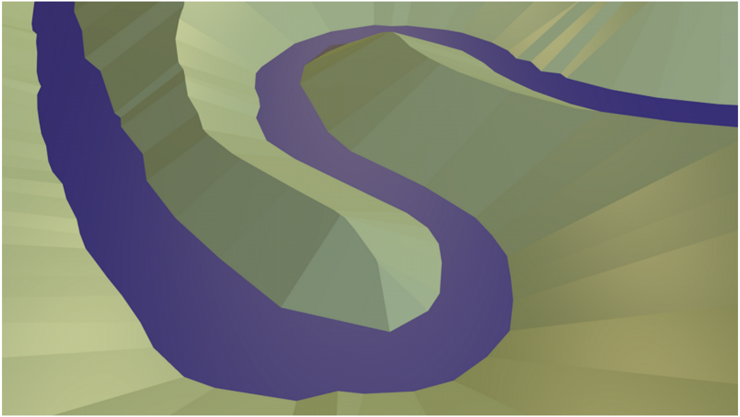


image credit: Stefan Huber

Applications: GIS



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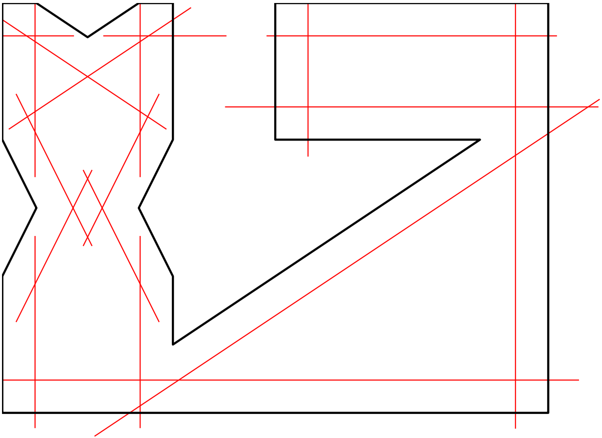
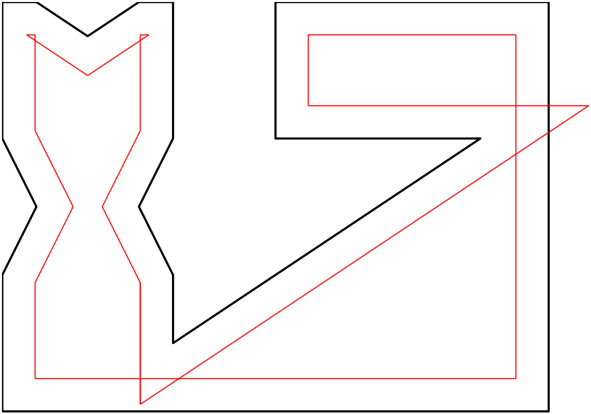
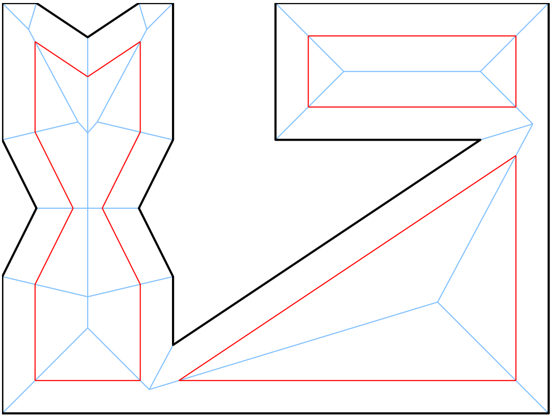
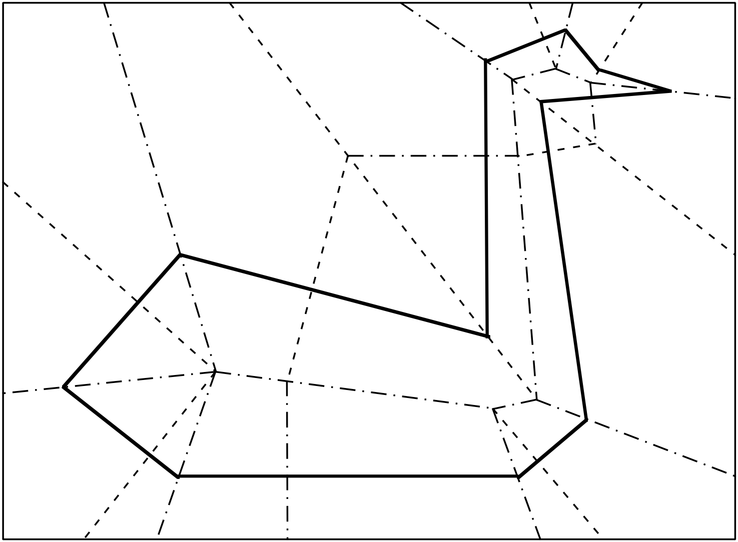




Applications: Offsetting



Applications: Cut and Fold



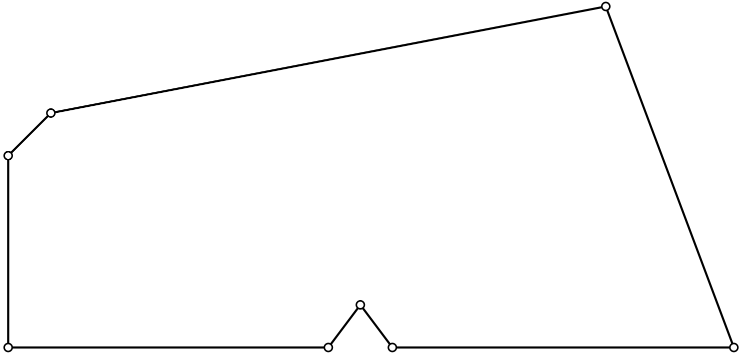
[DDL98]

image credit: Erik Demaine

APPLICATIONS: AND MUCH MORE

- Design of Pop-Up cards [Sugi13].
- Shape reconstruction and contour interpolation [OPC96].
- Area collapsing in geographic maps and centerlines of roads [HS08].

- Common approach: simulate the wavefront propagation.
- Problem: When will the next event happen, and what is it?
- If we solve this, we can incrementally construct the SK.



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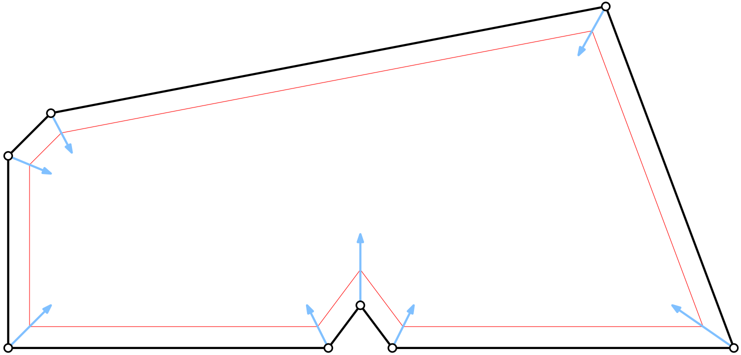

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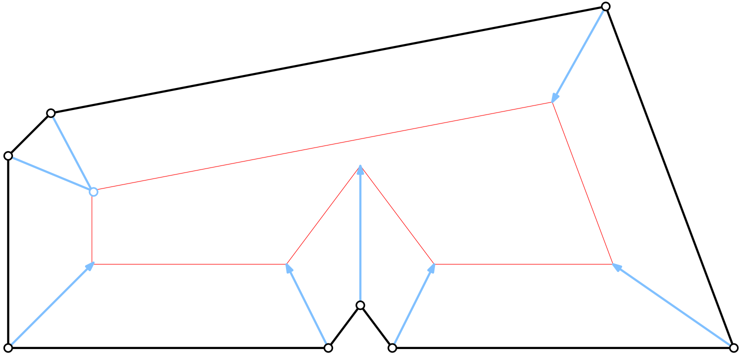

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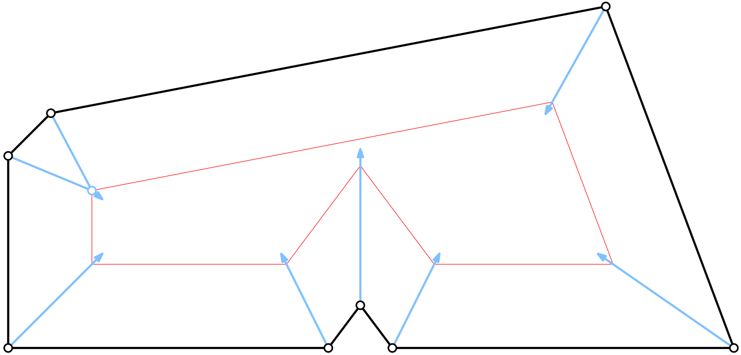

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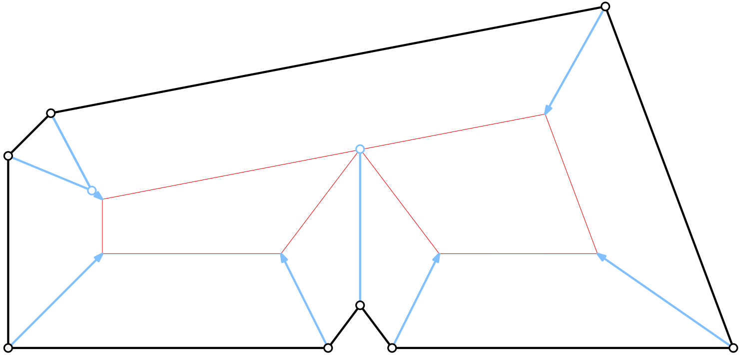

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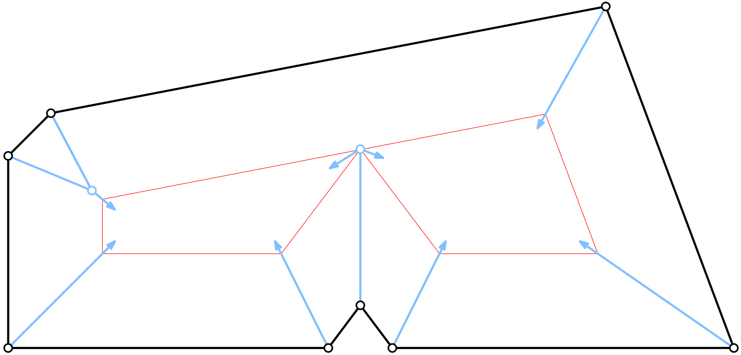

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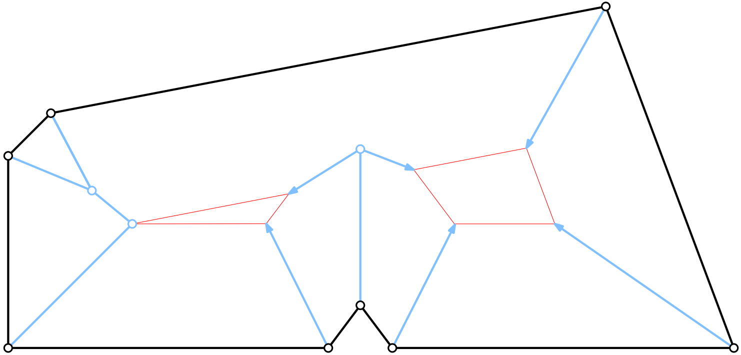

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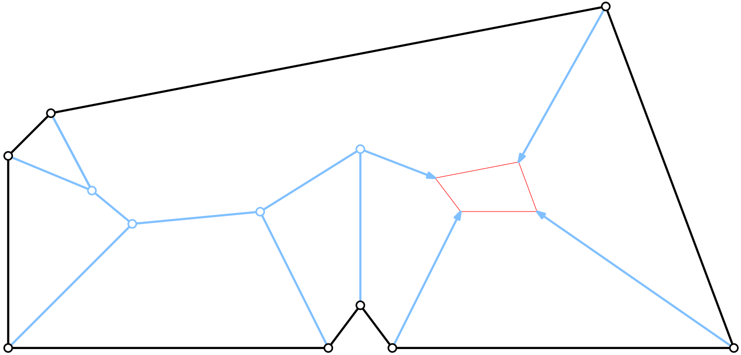

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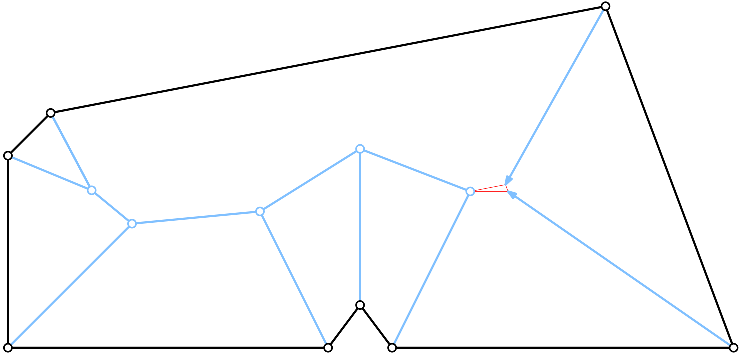

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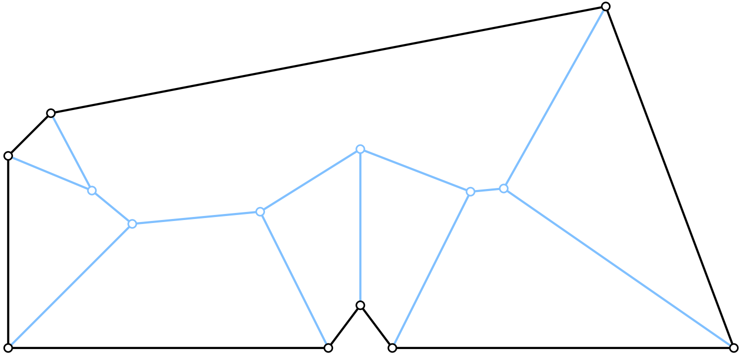
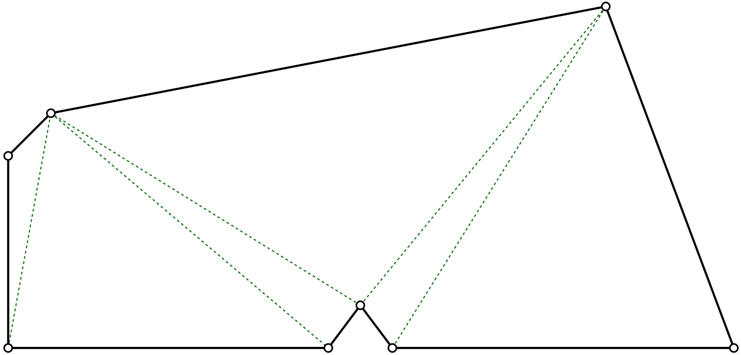


Triangulation-based Algorithm

- Aichholzer, Aurenhammer [AA98].
- Maintain a kinetic triangulation of the points of the plane not yet visited.



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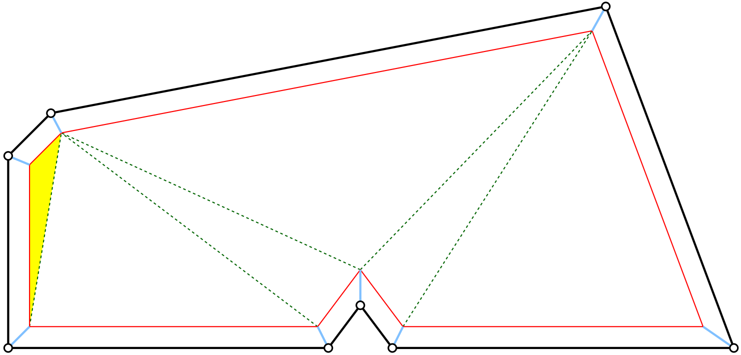

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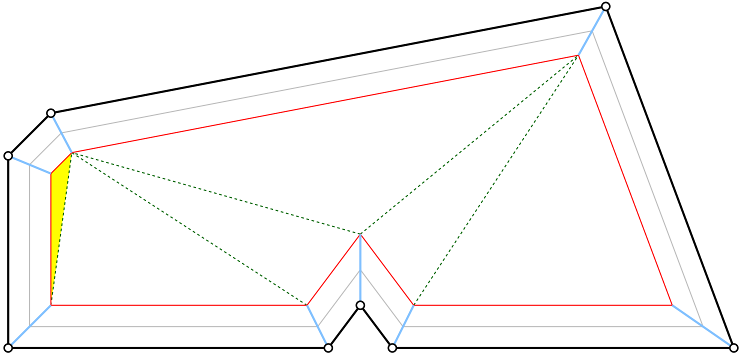

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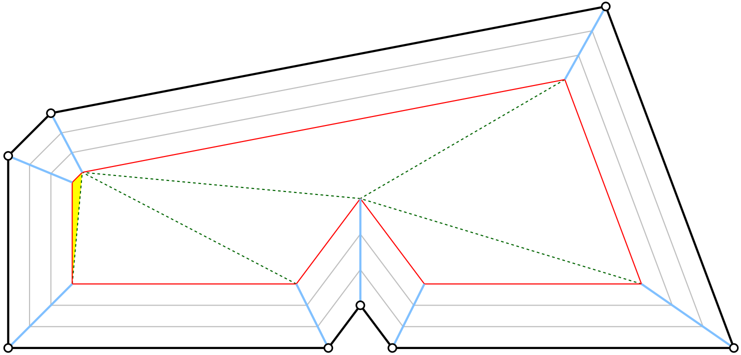

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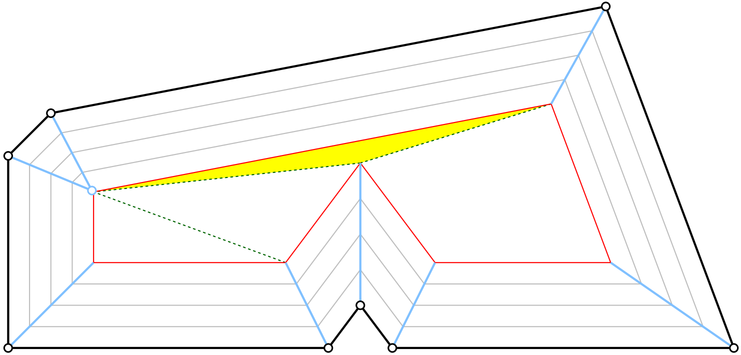

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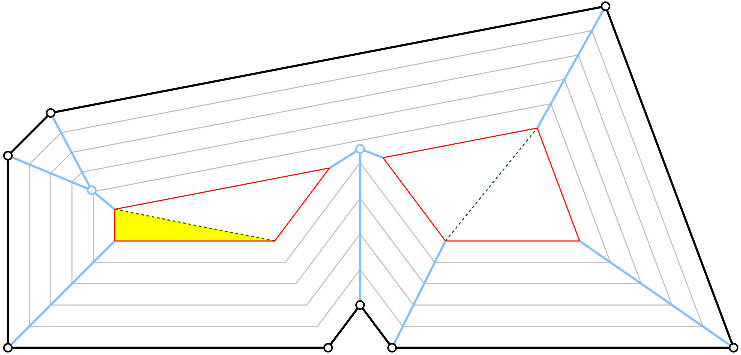

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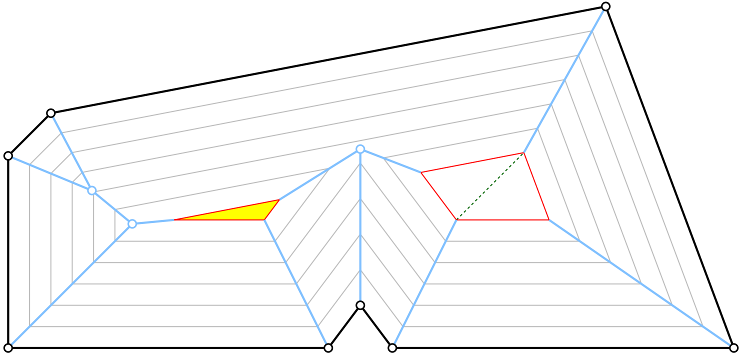

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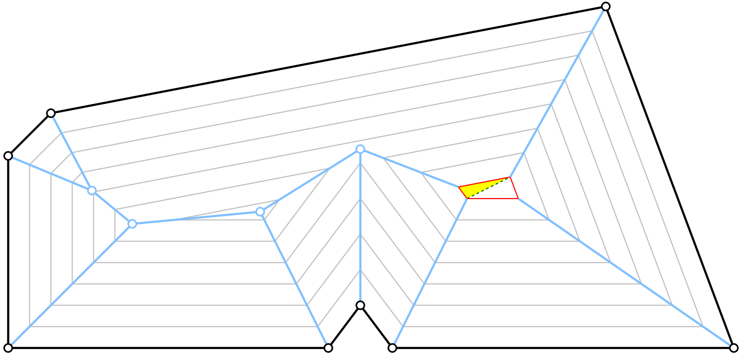

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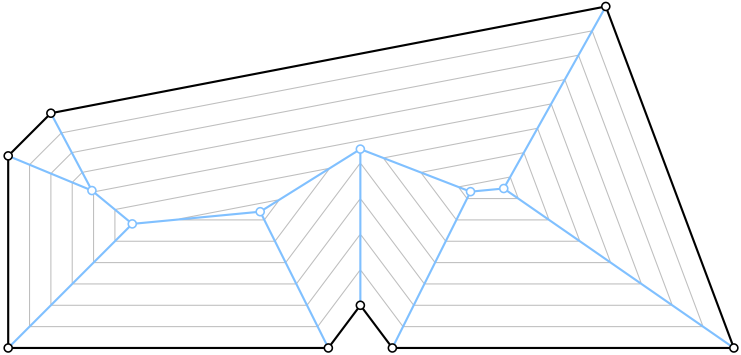

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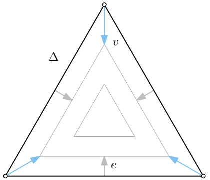

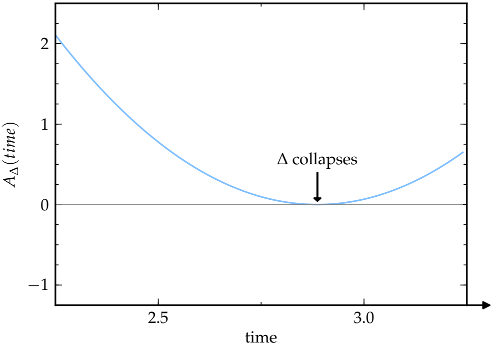

Triangulation-based Algorithm

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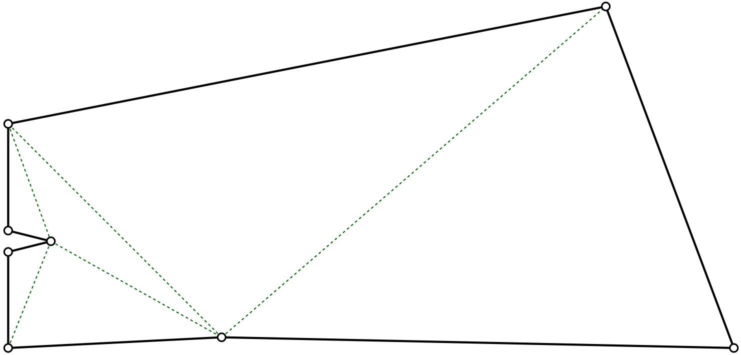
- *Collapsing triangles witness edge and split events.*

- Compute collapse times of triangles.
- Maintain a priority queue of collapses.
- On events, update triangulation and priority queue as required.

- *We can always easily find the next event, and thus compute the straight skeleton.*

- Caveat: Not all collapses witness changes in the wavefront topology.



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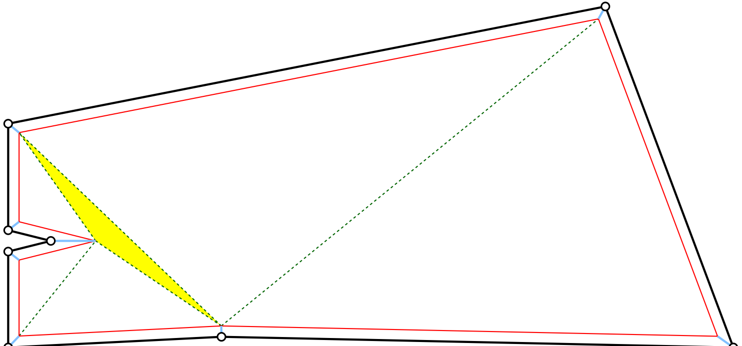

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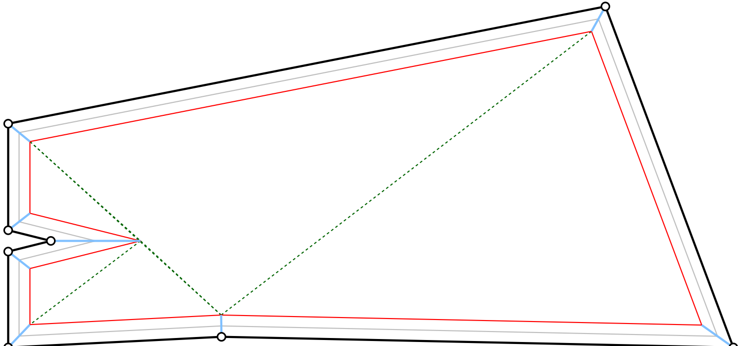

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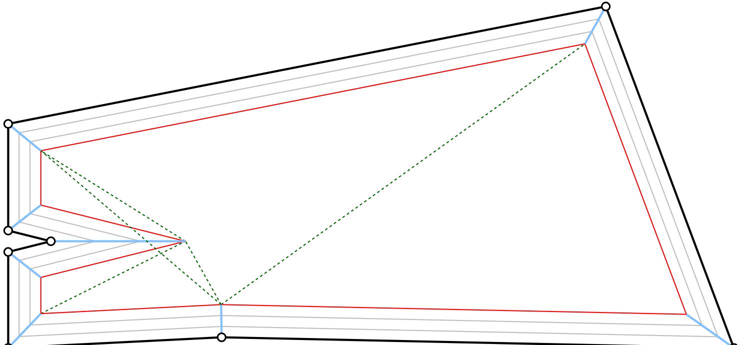

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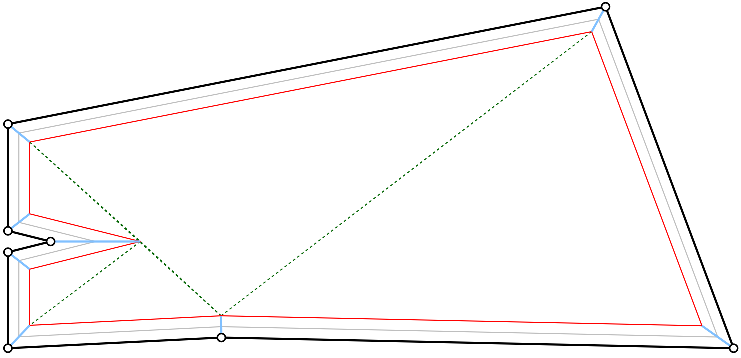

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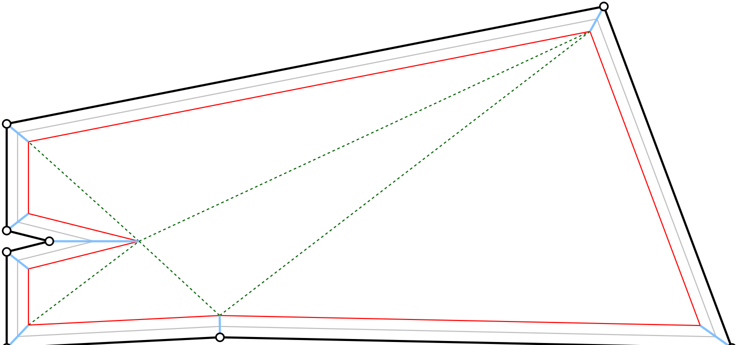

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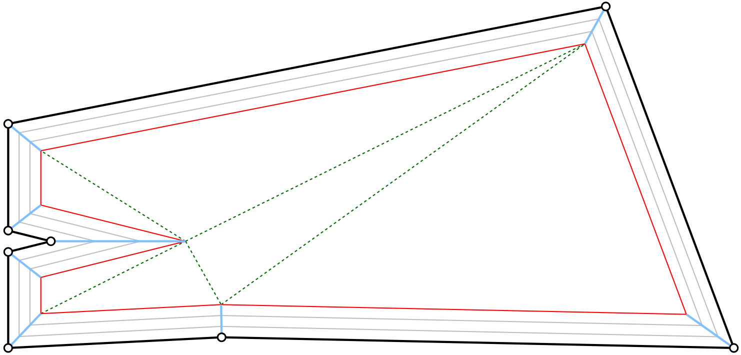

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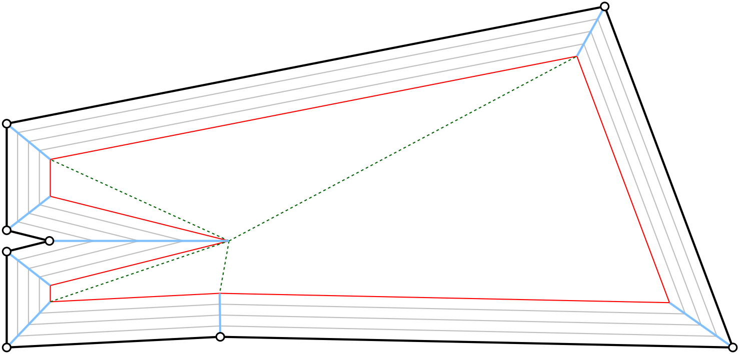

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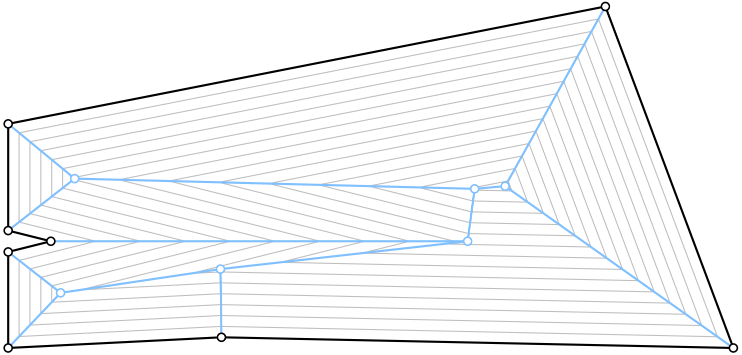
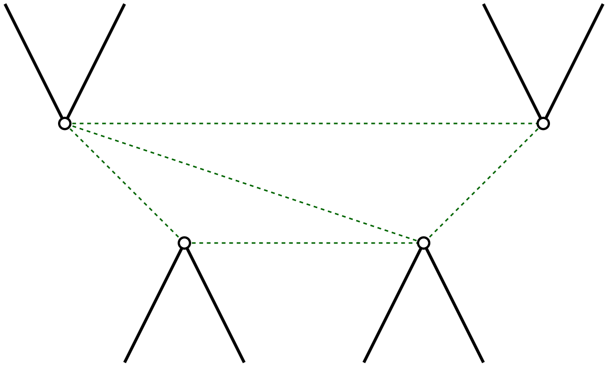


CONTRIBUTION [PHH12]

- We have implemented this algorithm.
- We filled in gaps in the description of the algorithm.
- The algorithm does not always work when input is not in general position. We have identified and corrected these flaws.
- We have run extensive tests using this code.

FLIP-EVENT LOOPS

- Without general position, this algorithm can end up in infinite loops.



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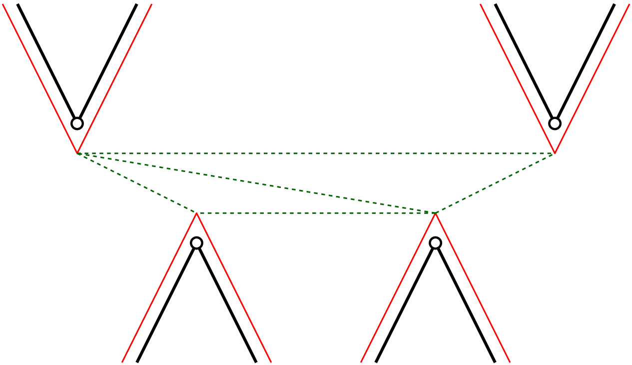

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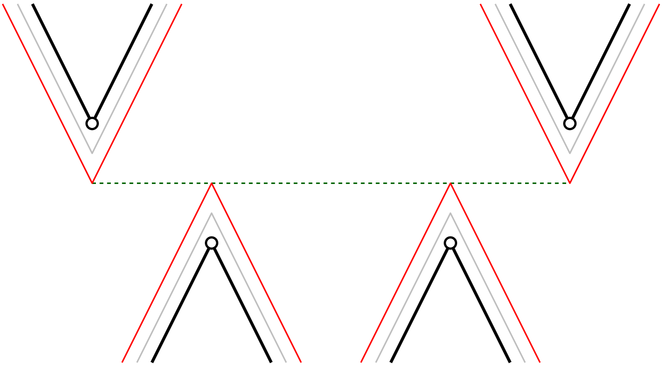

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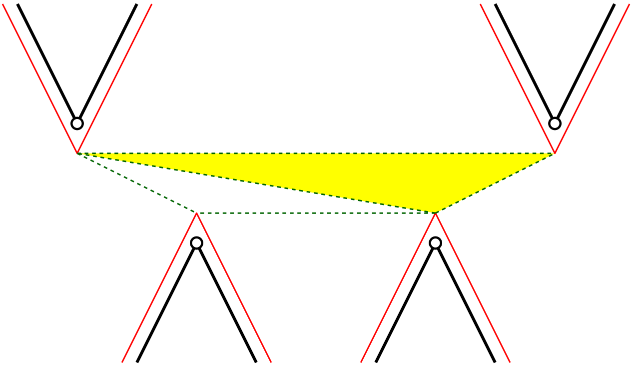

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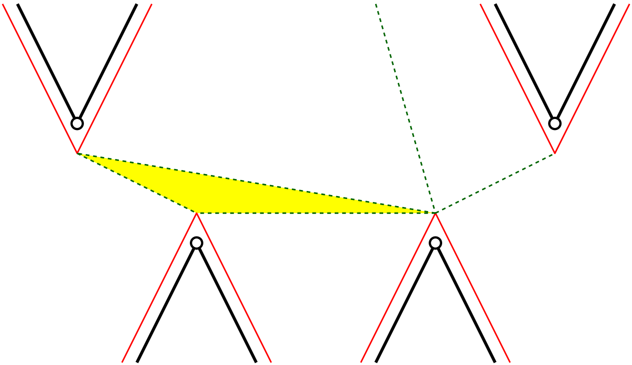

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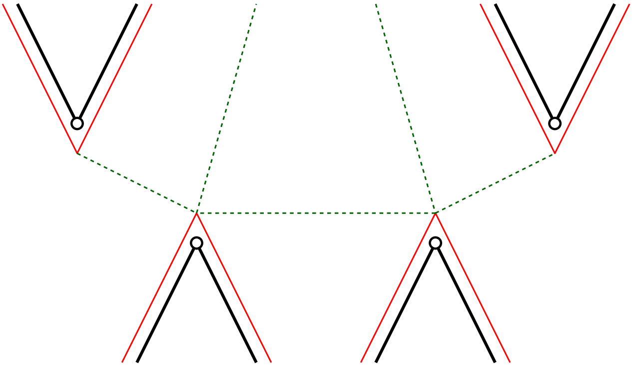

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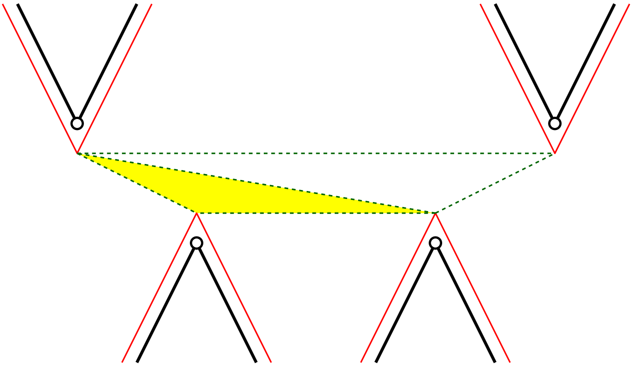

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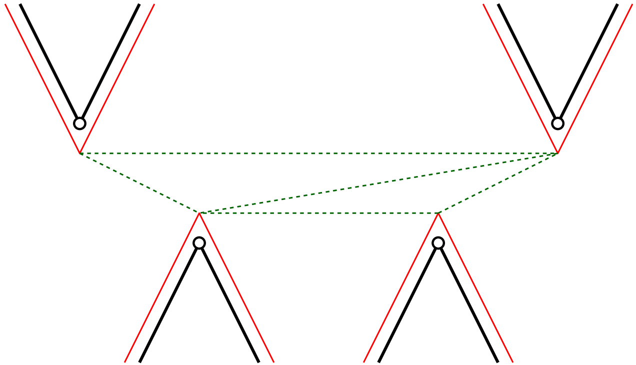

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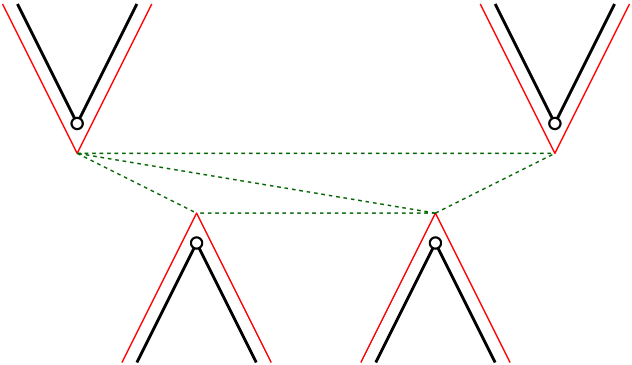

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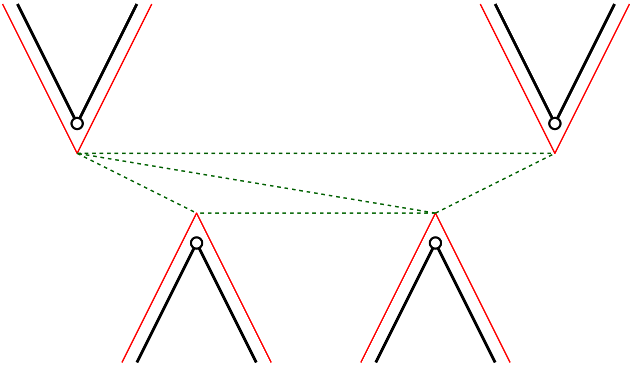


- This is not a result of inexact floating point operations. The same can happen with exact arithmetic!

AVOIDING FLIP-EVENT LOOPS WITH EGC

If we had exact arithmetic operations, the following would work:

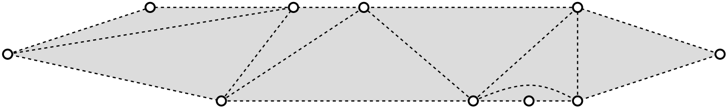
- First, pick the non-flip event $\rightarrow$ reduces triangles
- If only flip events are left, pick the one with the longest edge to flip $\rightarrow$ reduces longest edge (count or length)

Detecting flip-event loops

- Keep a history of flip events $\langle e_1, e_2, \ldots \rangle$ where each $e_i = (t_i, \Delta_i)$.
- This history can be cleared when we encounter an edge or split event.
- If we encounter a flip event a second time, we may be in a flip-event loop.

Brief outline:

- Identify the polygon P which has collapsed to a straight line.
- Retriangulate P and its neighborhood.



HANDLING FLIP-EVENT LOOPS

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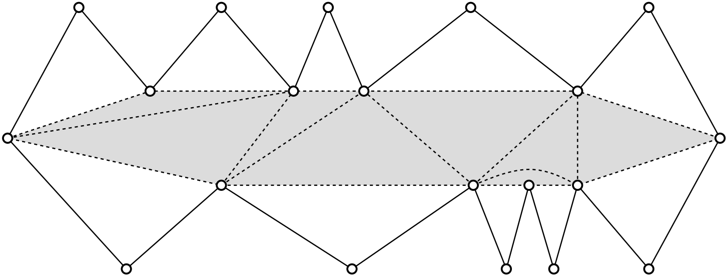

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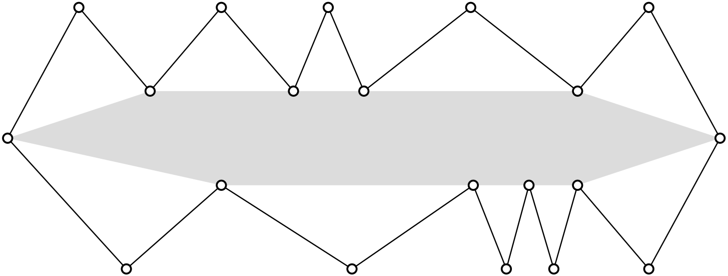

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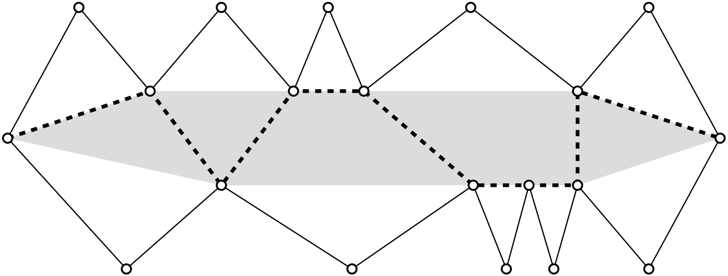

Brief outline:

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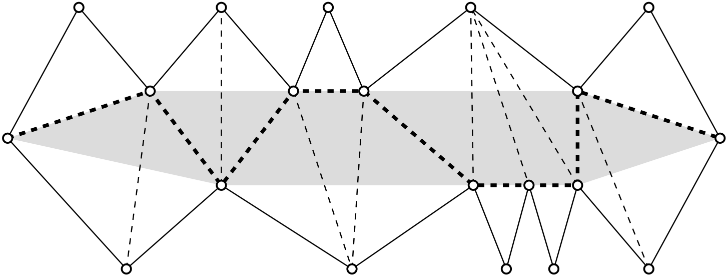

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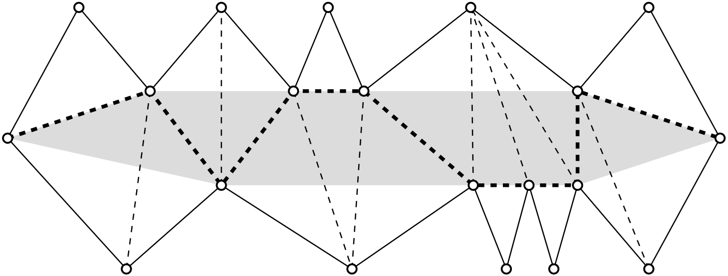
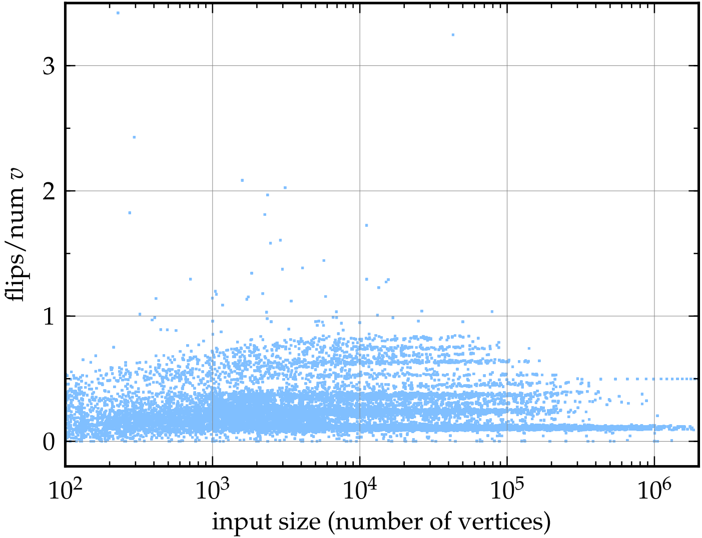


- This approach is also applicable to kinetic triangulations in other algorithms [MHH12].

NUMBER OF FLIP EVENTS

- $\mathcal{O}(n^3)$ is the best known upper bound on the number of flip events,
- No input is known that results in more than quadratically many flip events.
- It turns out that for *practical data* the number of flip events is very linear.

Number of Flip Events, II



Performance observations

	theoretical worst case		practical	
	runtime	space	runtime	space
E&E[1]	$\mathcal{O}(n^{17/11+\epsilon})$	$\mathcal{O}(n^{17/11+\epsilon})$	N/A	
CGAL[2]	$\mathcal{O}(n^2 \log n)$	$\mathcal{O}(n^2)$	$\mathcal{O}(n^2 \log n)$	$\mathcal{O}(n^2)$
Bone[3]	$\mathcal{O}(n^2 \log n)$	$\mathcal{O}(n)$	$\mathcal{O}(n \log n)$	$\mathcal{O}(n)$
Surfer[4]	$\mathcal{O}(n^3 \log n)$	$\mathcal{O}(n)$	$\mathcal{O}(n \log n)$	$\mathcal{O}(n)$

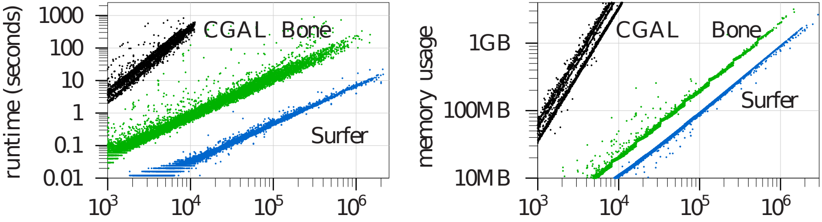
[1] Eppstein and Erickson [EE99]

[2] F. Cacciola, submission to CGAL, 2004

[3] Huber and Held [HH10]

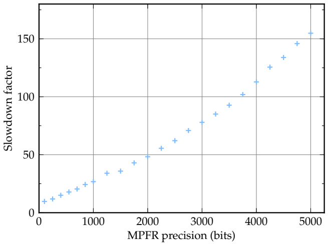
[4] Palfrader et al. [PHH12], based on Aichholzer and Aurenhammer [AA98]

Runtime tests

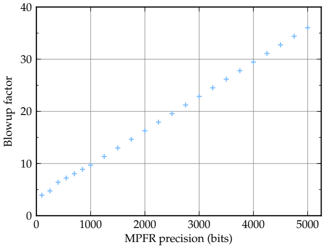


Runtime and memory usage behavior of CGAL, Bone, and Surfer for inputs of different sizes.
Bone and Surfer use their IEEE 754 double precision backend.

MPFR



slowdown



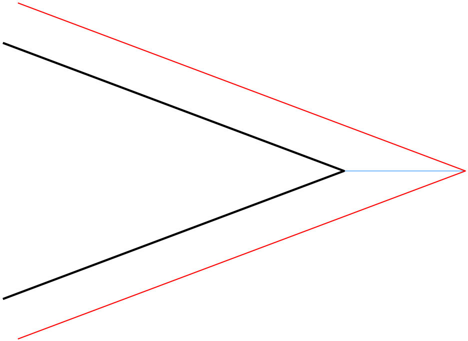
blowup

SUMMARY

- We have implemented Aichholzer and Aurenhammer's algorithm from 1998, filling in details in the algorithm description.
- We fixed real problems that arise in the absence of general position.
- Our approach to handling flip events has wider applications.
- The implementation runs in $\mathcal{O}(n \log n)$ time for *real-world data*. The number of flip events is linear in practice.
- It is industrial-strength, having been tested on tens of thousands of inputs.
- It is the fastest straight skeleton construction code to date, handling millions of vertices in mere seconds.

Future Work: Mitered Reflex Vertices

- The current definition causes fast moving vertices for angles approaching $2 \cdot \pi$.



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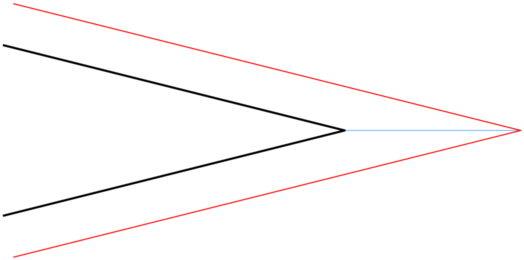

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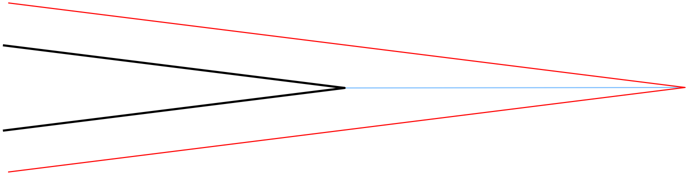

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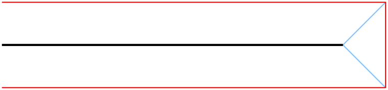

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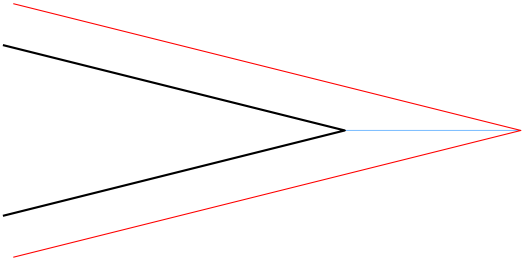
- The current definition causes fast moving vertices for angles approaching $2 \cdot \pi$.
- Investigate and implement some kind of restricted miters.



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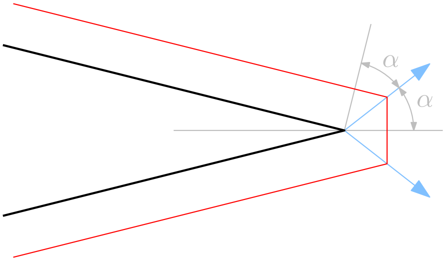


FUTURE WORK: UPPER BOUND

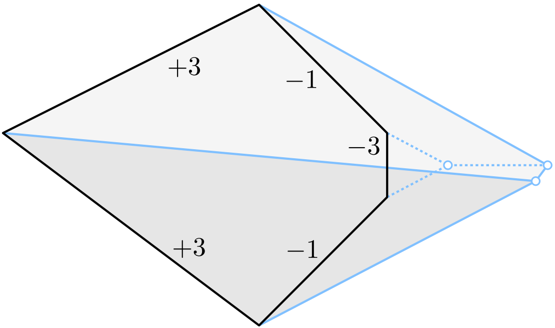
- $\mathcal{O}(n^3)$ is the best known upper bound on the number of flip events.

Future Work: Upper bound

- $\mathcal{O}(n^3)$ is the best known upper bound on the number of flip events.
- But Rubin showed $\mathcal{O}(n^{2+\epsilon})$ for kinetic Delaunay Triangulations where vertices move at unit speed [Rubin13].
- Can we transfer this result?

FUTURE WORK: WEIGHTED STRAIGHT SKELETON

- Weighted $\mathcal{SK}$: Edges move at different speeds, maybe even negative speeds.
- Which of the properties of the straight skeleton (planarity, tree structure, faces are monotone) carry over to weighted straight skeletons [BHHKP13]?



Thank you for your attention.

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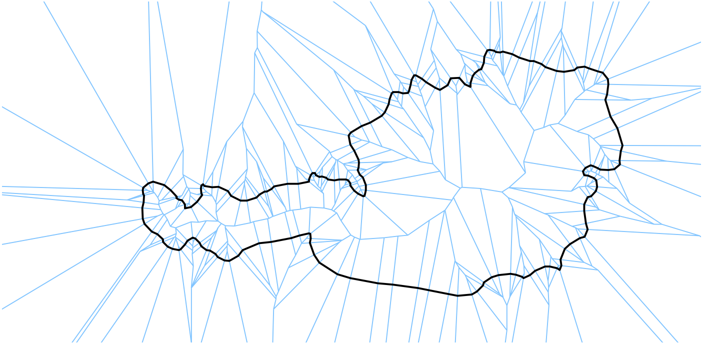
- Willi Mann, Martin Held, Stefan Huber, ”Computing Motorcycle Graphs Based on Kinetic Triangulations”, Proceedings of the 24^{th} Canadian Conference on Computational Geometry (CCCG 2012)
- Therese Biedl, Martin Held, Stefan Huber, Dominik Kaaser, Peter Palfrader, ”Weighted Straight Skeletons In the Plane”, Proceedings of the 25^{th} Canadian Conference on Computational Geometry (CCCG 2013)

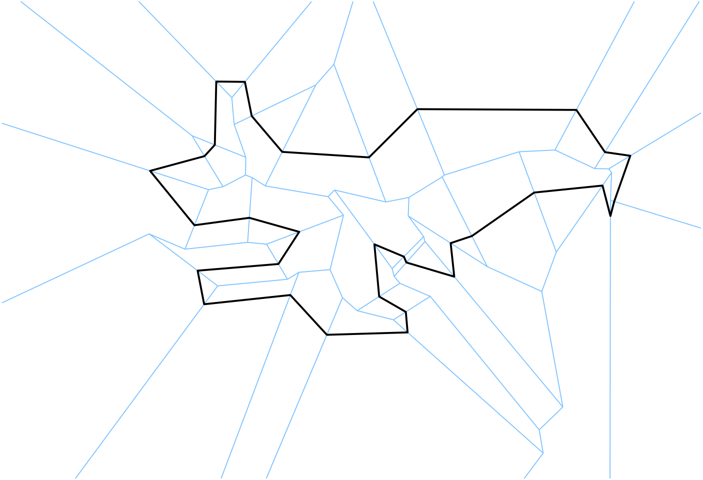
References II

- Natan Rubin, ”On Kinetic Delaunay Triangulations; A Near Quadratic Bound for Unit Speed Motions” Accepted to 54^{th} Annual IEEE Symposium on Foundations of Computer Science (FOCS 2013)

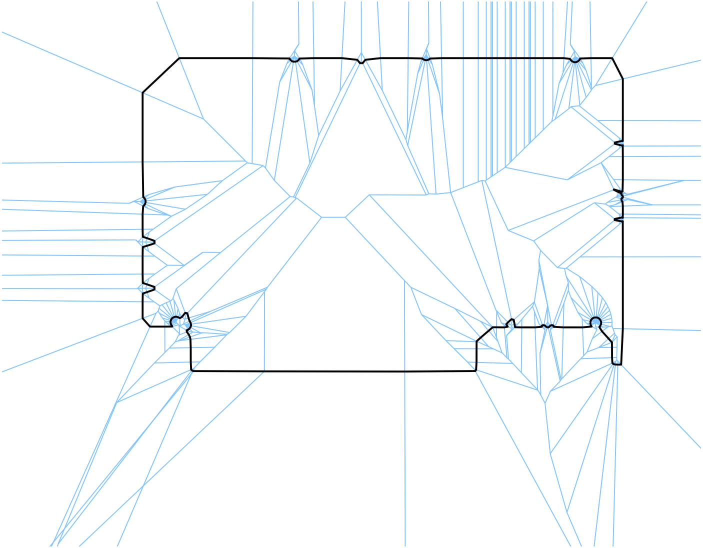
GALLERY: BORDERS OF AUSTRIA



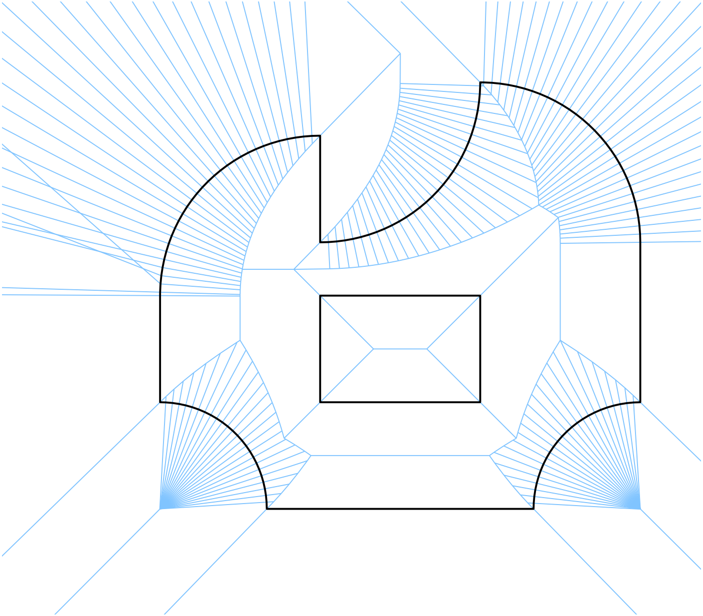
GALLERY: RANDOM POLYGON



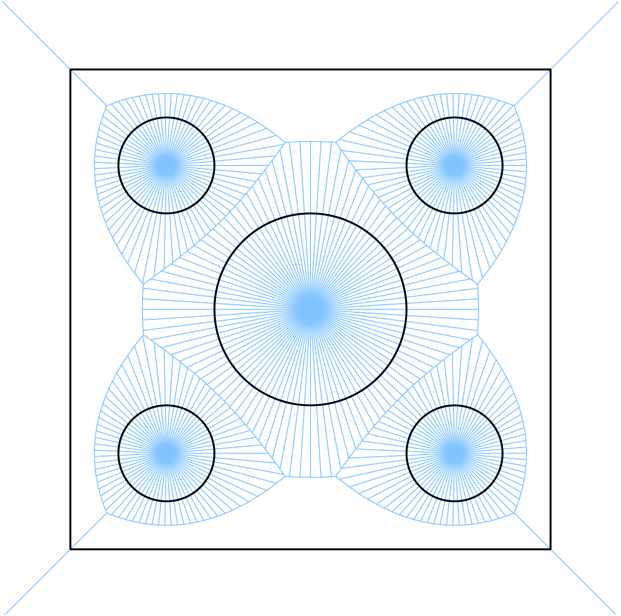
GALLERY: PCB



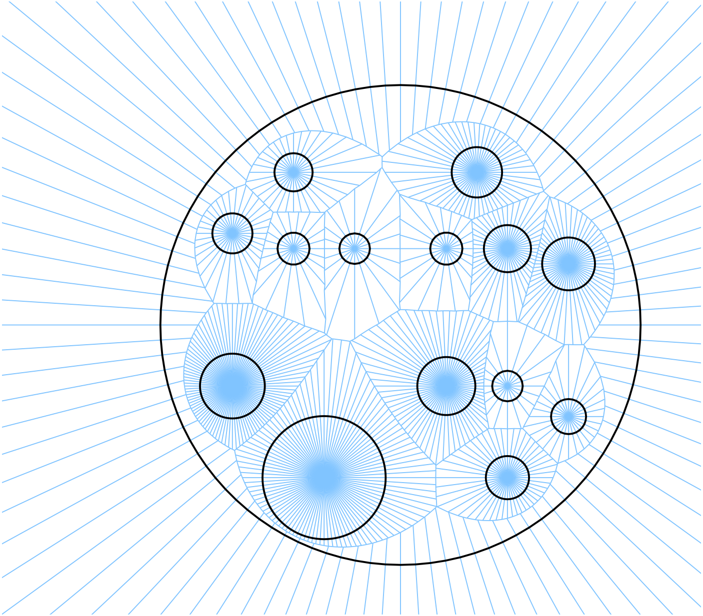
GALLERY: POLYGON WITH HOLE



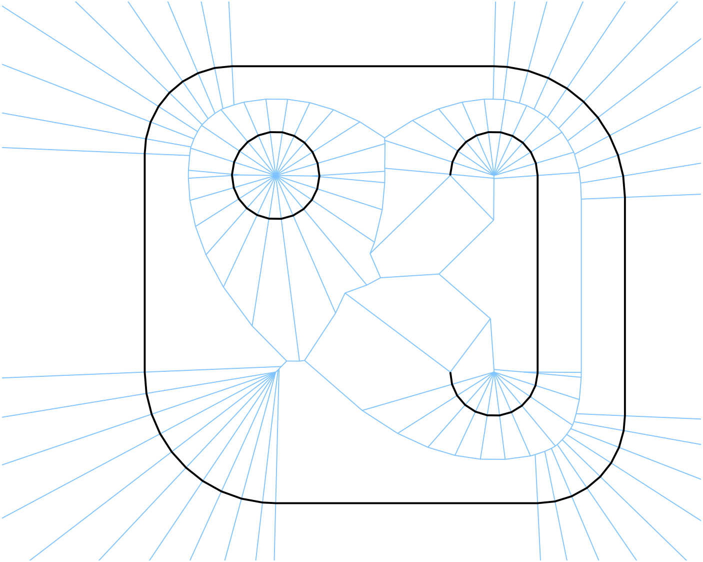
Gallery: Circular Holes



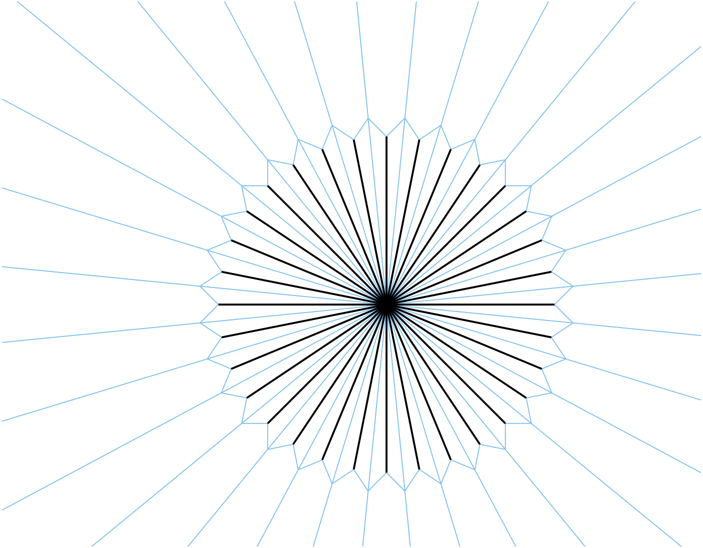
Gallery: More Holes



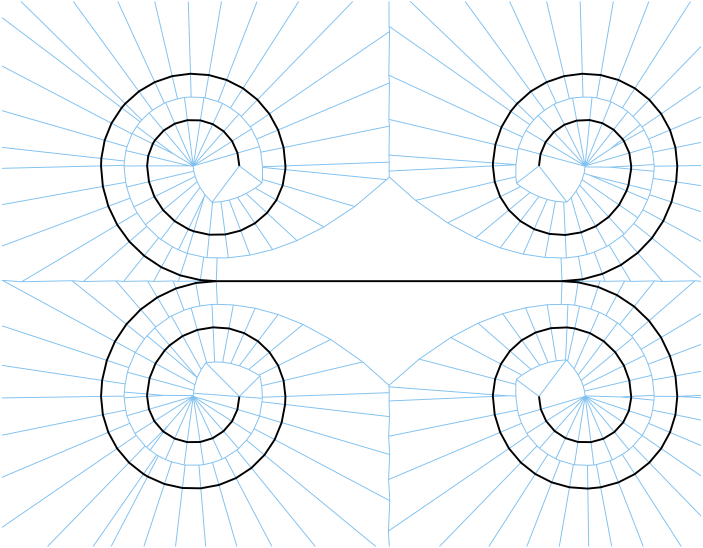
Gallery: Almost Polygon



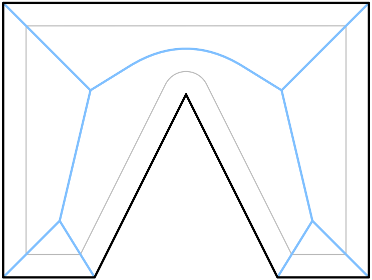
GALLERY: STAR

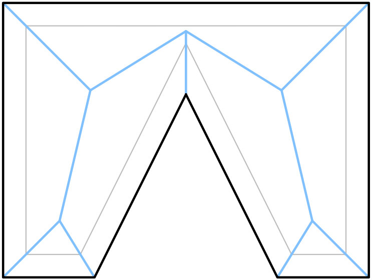


Gallery: Spirals

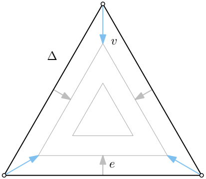


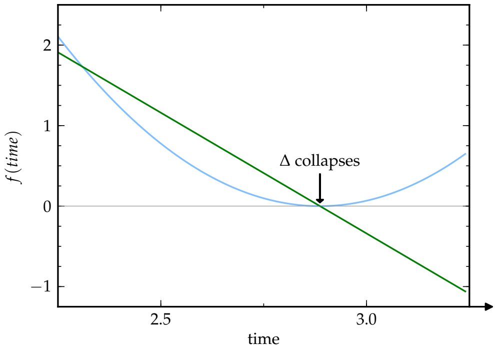
MEDIAL AXIS VS. SK


VD-based MA

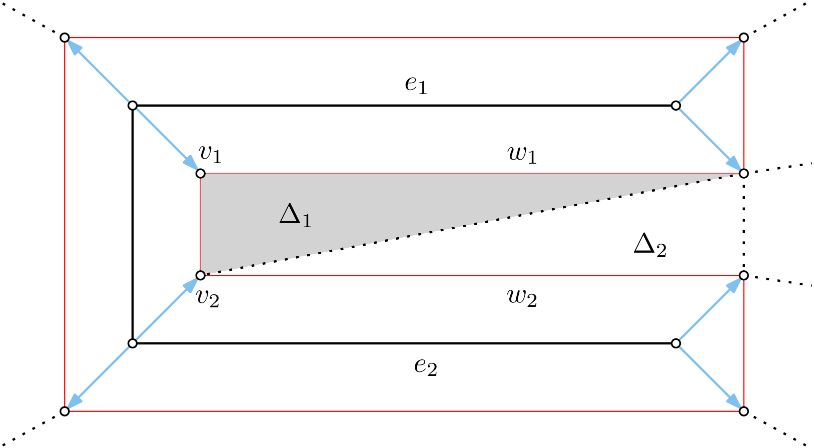

SK

Alternate Computation



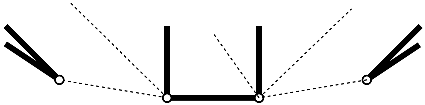


Infinitely fast vertices



TRIANGULATING

- Triangulate the convex hull.
- Unfortunately the convex hull changes with time, and it matters.



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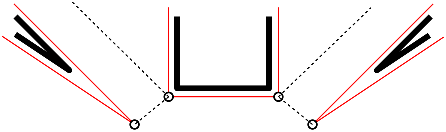

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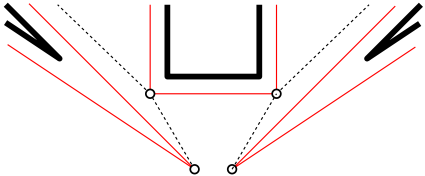

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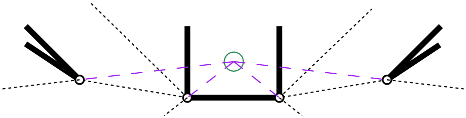

- We need to update the triangulation at some point before this happens, but how?

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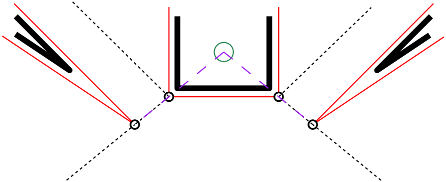

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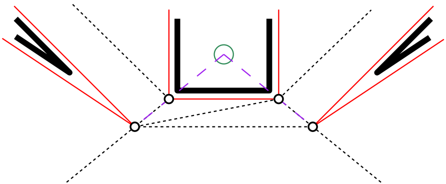

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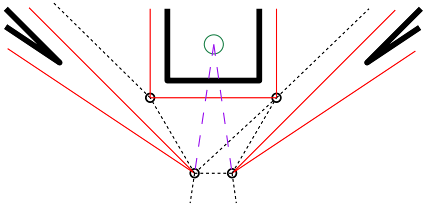

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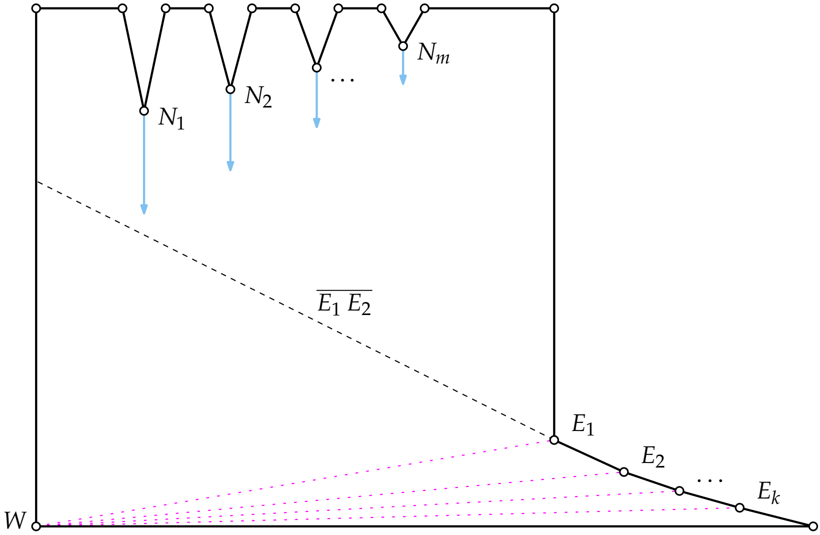
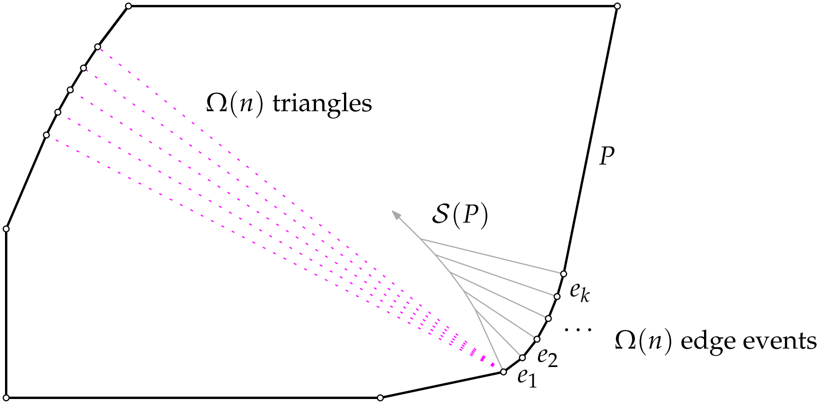
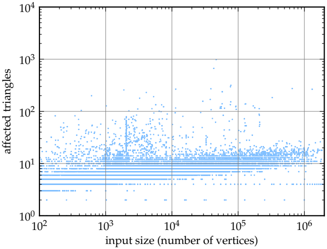
Ω FOR FLIP EVENTS



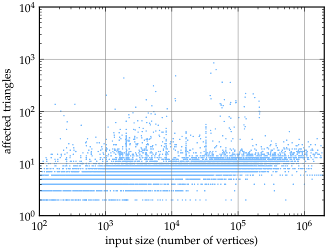
Ω FOR NON-FLIP EVENTS



Affected Triangles, Max

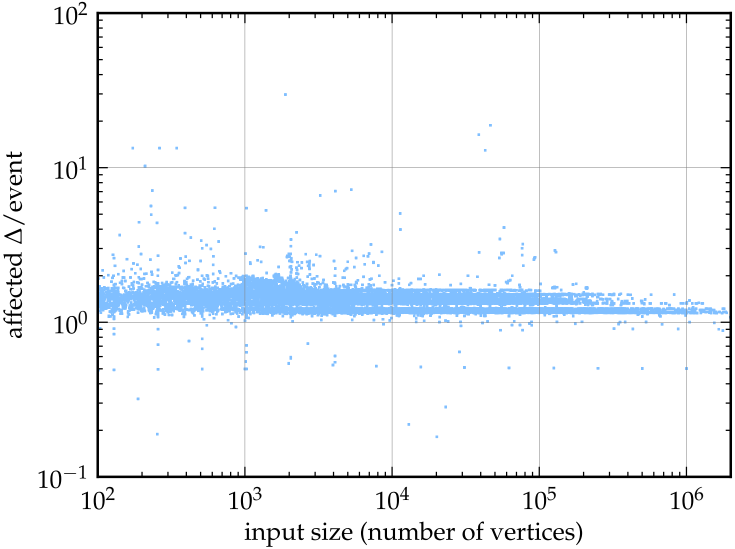


in edge events



in split events

Affected Triangles, Avg



Time spent, phases